

MODELLING OPTIMAL STRATEGIES FOR DETERIORATING INVENTORY SYSTEMS UNDER DIFFERENT SCENARIOS

A Thesis submitted to Gujarat Technological University

for the Award of

Doctor of Philosophy

in

Science- Mathematics

by

Dharmesh Kathadbhai Katariya

Enrollment No. 189999906002

under supervision of

Dr. Kunal Tarunkumar Shukla

Assistant Professor

Vishwakarma Government Engineering College, Chandkheda,
Ahmedabad



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AHMEDABAD

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ABSTRACT

Product deterioration, a genuine phenomenon, poses an obstacle to numerous inventory management systems. The shelf life of a product and its physical condition is expressed by its deterioration rate. An extremely fascinating aspect of managing inventory is mathematical modelling for products that are deteriorating. The pace of deterioration varies depending on the product's characteristics or service. The deterioration behaviour for an inventory model can belong to different kinds: the models with a fixed lifetime of the product, the models with an age-dependent deterioration rate, i.e., a probabilistic distributed life time, and the models with a time- or stock-based or constant deterioration rate. *Our research carried out three different scenarios for deteriorating inventory models: scenario 1: the inventory models of the "new and buyback used products" concept; scenario 2: the inventory models with carbon emissions and green investments; and scenario 3: the inventory models that incorporated freshness and greening efforts for perishable products.*

The purchasing behaviour of consumers has changed nowadays. Consumers not only prefer the newly launched product on the market, but they are also interested in purchasing used, refurbished, recycled, or repaired products with price discounts. They are also concerned about environmental issues and prefer to purchase goods from manufacturers or retailers with a green reputation. For this reason, a lot of businesses have started gathering used products that buyers throw away. With this in mind, we created deteriorating inventory models without and with shortages as retailer points for both newly released products and buyback used products and optimized the ordering quantity of new products, buyback quantity of used products, and replenishment cycle time such that the retailer's profit is maximized.

Controlling carbon emissions has been the primary objective for nations since the emission of carbon causes numerous problems in the global ecosystem. Ordering, production setup, purchasing, storage, impact on the environment, transport, and other inventory system activities all result in the emission of carbon. The management of deteriorating inventory with green technology investment has been one of the areas that contribute to mitigating carbon emissions. Our research work focused on designing sustainable inventory models to

minimize carbon emissions using green investments and applying various carbon policies and trade credit payment systems to the demand of deteriorating products depending on selling price, green investment cost, and their promotion, resulting in a total profit maximized and supply chain costs minimized.

Nowadays, consumers who are concerned about their health prefer and expect nutritional and fresh sustainable products. Product freshness is an essential component of its quality, and as a result, the choice of purchase for consumers depends on the freshness of the green products. Due to the effect of physical deterioration and quality degradation of the product; the product loses its originality continuously, so market demand decreases and hence retailer or producers offers the price discount or markdown strategy to stimulate the demand. Greening efforts are the action taken to minimize the impact trade has on the ecosystem and ensure sustainable products. Taking into account all of these factors, developed the inventory models with the demand is a function of the selling price, age of products (freshness), and greening efforts for deteriorating perishable products and optimize the retailer's or producer's profit is maximize.

The objective of the research work is to maximize the total profit or minimize the total cost of the retailer, producer, or manufacturer at the optimal value of decision variables. Models are validated through numerical examples, sensitivity analysis of parameters, and graphical demonstrations of objective functions, and managerial insights are derived from the analysis. Some concluding remarks, along with future scopes, are discussed in each chapter.

Dedicated
to
My Late Father
Kathadbhai Mayabhai Katariya
&
My Late Brother
Alpesh Kathadbhai Katariya

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Dharmesh Kathadbhai Katariya

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CHAPTER-1

Introduction

1.1 Operations research

“Operations Research is applied decisions theory. It uses any scientific, mathematical or logical means to attempt to cope with problems that confront the executive, when he tries to achieve a thorough going rationally in dealing with his decision problem.”

-D.W. Miller and M.K. Starr

An operation research (OR) is a discipline of mathematics that gives management a rationale for making decisions quickly and efficiently. Operations research is a mathematical approach to issue assessment and decision-making, and it is frequently referred to as management science or decision science. Operations research originated during the second world war, making it a *war baby*. After the second world war, the armed forces in Britain requested the support of a group of specialists to examine the issue of national defence. The issue with regard to resource optimization was presented to the specialists, and they were tasked with coming up with an answer that is feasible. The "Linear Programming" method was successful in handling the war issue. As the name implies, *operations* refer to wartime challenges, and *research* focuses on the creation of new techniques. When the War was over, the military teams' achievements drew the attention of Industrial Managers, who were looking for answers to their challenging executive-type difficulties. American mathematician George B. Dantzing[1] created the first mathematical method in this discipline, known as the Simplex Method of Linear Programming, in 1947. Recent years have seen a significant increase in the use of operations research in a variety of technical applications as well as management, supply chain management, and managing manufacturing. It has expanded like a large optimization

branch. Operations research has supported organisations and industries in different fields like inventory, replenishment systems, the best place for storage and dimensions, marketing policies, both public and private sectors on energy policy, defence, medical services, town planning and their maintenance; transit challenges, water resource management, the world wide web, the aviation industry, global financial institutions, and many others areas.

1.2 Inventory Management

Inventory management has been one of the most extensively researched fields in operations research. The focus of operations research is usually on investigating and analyzing inventory management systems. The products and materials that an organization maintains on hand with the intention of retailing, manufacturing, or using them are referred to as *inventory*. Inventory is an aspect of financial resources, and the main focus of consideration is costs related to inventory. Several strategies for financial optimisation have been established for controlling costs. The real-time availability of commodities, resources, or services is also known as inventory management. This allows for well-coordinated corporate operations to meet both the demands of today and the needs of tomorrow. Inventory management aims to increase revenues while requiring the least amount of inventory and without compromising client retention standards. This raises the following fundamental inventory management questions in the study's perspective.

- How many units should be requested?
- When should place an order?
- How much of the reserved commodities should be kept?

Over the courses of several years, researchers have dedicated their efforts to developing a framework that aims to determine the optimal order quantity and replenishment cycle times, with the ultimate goal of minimizing inventory costs. As a result, the next part attempts to comprehend the many inventory types and costs associated with inventory management, followed by basic models of economic order quantity and economic production quantity that give the optimal inventory cost for any item according to the discussion. The relevant types of inventory system mentioned in next section.

1.2.1 Inventory types

Inventory can be divided into two categories- direct inventory and indirect inventory.

- Direct inventory is that which is utilised in the process of producing the product. It is further classified into the types listed below.
 - **Raw material inventories:** Raw material inventory is stock that has not yet been utilised for production. Raw material inventories serve to guarantee that firms have enough of the materials they require to continue operating.
 - **Work-in-progress inventories:** It refers to partially completed products in any manufacturing cycle. Work-in-progress inventory is a significant component of a manufacturer's financial report and a key sign of the supply chain's stability. The accounting of work-in-progress inventory helps a business estimate the worth of its stock.
 - **Finished-goods inventory:** The total quantity of produced items that are accessible, in stock, and ready for purchase by suppliers, retailers, and consumers is referred to as finished goods inventory. In order to validate the accuracy of financial information for the present and following cycle time, finished goods inventory is excellent for monitoring manufacturing and work-in-progress inventory.
 - **Spare parts inventory:** A spare part inventory is a collection of stock goods used by mechanics instead of faulty components.
- Indirect inventory does not affect finished products, despite the fact that it is important for production. Indirect inventory is classified as follows:
 - **Fluctuation inventory:** This serves as a balance between marketing and manufacturing. The fluctuation inventory is a reserve stock that is retained to maintain variations in demand and delivery times that have an impact on the manufacturing of products.
 - **Anticipation inventory:** Anticipation inventory is reserve stock according to expected demand in the future. It is kept on hand in anticipation of future need, such as seasonal peak sales, plant shutdowns, festival-related times, and so on.
 - **Transportation inventory:** Transportation inventory is a consequence of the need to move products from one location to elsewhere.
 - **Decoupling inventory:** Decoupling inventory principles are utilised in the industry to ensure the manufacturing process does not halt. An organisation maintains an inventory

of unfinished objects at every phase of manufacturing, which may be utilised in the event of equipment or procedure disruption or failure.

1.2.2 Essential terminologies used in inventory modelling

- **Demand:** Demand is the amount of a commodity that is required at a certain point in time. It can be noticed in most cases with regard to time, selling price, quantity, and/or environmental factors. The demand for a commodity can be classified as deterministic demand or probabilistic demand. Demand is referred to as deterministic when a precise or realistic quantity of the commodity is determined in advance. Furthermore, if the demand over a particular duration is uncertain yet the initial scenario can be stated in terms of a distribution of probabilities, it is said to be probabilistic or stochastic demand.
- **Cycle time:** The interval between successive subsequent replenishments is known as the cycle time. It may be measured as, (i) Continuous review or (ii) Periodic review.
 - (i) **Continuous review:** In this case, an order of a particular size is placed whenever the amount of stock turns a pre-specified level (known as reorder level). This is also known as the fixed order level or the two-bin system.
 - (ii) **Periodic review:** In this case, orders are placed at regular intervals, and the amount of the order is determined by the quantity of inventory on hand as well as the number of orders that are in progress at the time of the review. Periodic review also known as fixed order interval system.
- **The planning horizon:** The planning horizon refers to the time frame during which a specific inventory level will be retained. It may be finite or infinite.
- **Lead time:** Lead time is the amount of time that passes between placing an order and having it fulfilled.
- **Reorder level:** A replenishment order is issued for a specified quantity when the inventory level meets a specific level, known as the reorder level (or reorder point). It anticipates that the inventory level is continually reviewed. The reorder level is determine as;
$$\text{Reorder level (ROL)} = (\text{Demand rate in unit during lead time}) (\text{lead time in time unit})$$
- **Optimal order quantity:** Optimal order quantity is the optimal replenishing order amount that ensures the overall cost of inventory throughout the specified time frame is minimized or the total profit is maximum.

- **Trade credit:** Trade credit is a debt with no interest which allows buyers to buy products with payment due at a later time without incurring any extra charges. The temporal extend of trade credit is commonly referred to as the trade credit period. The retailer has the opportunity to generate interest income by selling the products during the trade credit period. In the event that the retailer is unable to satisfy the account within the designated credit period, an interest charge will be imposed.
- **Deterioration:** Deterioration is the term used to describe decay, damage, or spoiling that prevents an item from being used for its intended function. Deterioration is a natural characteristic. The pace of deterioration varies depending on the good or service. Products like blood, vaccines, vegetables, fruits, dairy products, beverages, etc. that have the maximum possible expiration dates are known as perishable deteriorating products. The rate of decay may be static or decreasing due to preservation technology. The deteriorating products might be divided into two different types:
 - (i) **Instantaneous deterioration:** Instantaneous deterioration is the term for the deterioration that occurs as soon as the product is manufactured or when it first enters the inventory system.
 - (ii) **Non-instantaneous deterioration:** The term "non-instantaneous deterioration" refers to deterioration that starts to happen after a certain amount of time has passed since the product was produced or when it first reaches the system of inventory.
- **Price discount:** Discounted pricing is an incentive pricing approach that reduces the initial selling price of products to increase demand for a specific period. A price discount is the practice of offering a product at a reduced price as a percentage of its original selling price.
- **Markdown policy:** A markdown policy is intended to encourage sales and get stock clear of slow-moving or remaining inventory; as the season approaches an end, enhancing the overall profit or minimize the overall costs. The decision maker optimizes the product quantity and pricing of the product under the markdown policy such that the profit is maximized.
- **Green technology:** Green technology is an invention in science and technology that reduces emission levels, uses fewer fossil fuels and other natural resources, and makes products more usable through refurbishing or recycling.
- **Carbon Pricing:** A carbon price is a financial strategy that offers manufacturers or businesses a monetary indication and gives them the option of changing their

operations to reduce their carbon emissions or maintaining their emissions and being charged for those emissions. The carbon price encourages environmentally friendly innovations and economic growth, consequently generating new, low-carbon business models. Carbon tax, carbon cap and trade, carbon offset, carbon limit, are the different carbon pricing policies.

- **Carbon tax:** For each unit of carbon emissions, businesses and industries are required of paying a certain amount, which is determined by the authorities and is known as a carbon tax. In other words, a tax known as a "carbon tax" is imposed on carbon emissions generated during the production of products and services. To avoid paying the tax, companies, and individuals will take actions to minimize their emissions, such as changing to renewable energy sources or using cutting-edge technology.
- **Carbon cap and trade:** According to the cap-and-trade policy, carbon emissions can be dealt on the carbon exchange; in other words, this policy imposes an enforceable restriction on emissions for all businesses that perform commercial activity and enables enterprises to purchase or sell carbon emission credits.

1.2.3 Various costs related to inventory modelling

- **Ordering cost (Set-up cost):** The cost paid while placing an order with a supplier is recognized as the ordering cost. It comprises of all management-related expenditures, such as transportation, product audits, administrative expenses, transaction charges, and other costs such as wages of procurement employees, phone calls, internet and computer costs, government taxes, stationery, etc. An item's setup cost, which includes both administrative and operational charges, is referred to as the ordering cost when it is manufactured privately. Usually, this cost is given as a cost per order or per setup.
- **Purchase (or Production) cost:** The actual price that must be paid at the stage of procurement of finished goods is referred to as the purchase cost, while the cost that must be incurred when producing goods is known as the cost of production. The production cost includes the price of the raw materials used to make the product, the salary paid to the labourers who make the product, and any other expenses related to the production process. It is unaffected by the size of the ordered quantity or the produced quantity.
- **Holding (Carrying) cost:** The cost involved with storing inventory is referred to as holding costs. The rent for the warehouse, maintenance cost, interest on the money,

preservation cost of the product, insurance, depreciation, goods and service tax, carrying charges, etc. are the parts of the holding cost.

- **Shortages cost or Stock out cost:** The capital lost from inventory that is no longer available for consumers to buy is known as stock out costs. The business loses revenue when a consumer can't buy a product because it is no longer available. This cost component is influenced by the amount of stock that is not provided to the consumer, not the source from which it is replenished.
- **Lost sale cost or Opportunity cost:** Opportunity cost is the price associated with a lost sale and is calculated as gross profit margin plus goodwill loss.
- **Transportation cost:** The cost associated with the delivery of inventory is called transportation cost. Transportation cost included the vehicle fuel cost and vehicle service cost.
- **Deterioration cost:** The cost generated as a result of inventory items degrading is known as the deterioration cost. The cost due to product decay, evaporation, obsolescence, damage, or spoilage is defined as deterioration cost.
- **Carbon emission cost:** Carbon emission cost is defined as mechanisms that put a specific price on greenhouse gas emissions, i.e., a price stated as a value per unit of greenhouse gas equivalence. Industries or organizations that pay a cost through a carbon tax, carbon offset, carbon cap-trade, or other environmental regulations are considered to have a carbon emission cost.
- **Green investment cost (Green finance):** Green investment cost or Green finance is a provision that can be used to describe capital expenditures made into initiatives and projects for environmentally friendly development, ecological products, and regulations that promote the development of an economy that is more ecologically.
- **Product unit cost:** The cost of procuring one unit for a business is known as the unit cost, which is the amount that vendors or buyers spend for one unit of the product.

1.2.4 Types of inventory models

An inventory model is a mathematical tool that supports businesses in figuring out the optimum level of inventories that should be retained in a manufacturing procedure, handling the ordering rate, calculating the number of products or raw materials to be stored, and monitoring the pattern of supply of products and supplies to ensure that customers receive continuous service without undergoing disruptions in delivery.

- **Economic order quantity (EOQ) or production quantity (EPQ) model:** The earliest and most prevalent methods for optimizing inventory is EOQ and/or EPQ modelling, which identifies the appropriate order (production) quantity to minimize overall inventory costs and maximize profit. Minimizing the cost of the product's acquisition, shipping, and holding can be done to determine the optimal number of order quantities(production quantity) and cycle time.
- **Sustainable inventory models:** Sustainable inventory modelling focuses on minimizing both social and ecological implications without influencing profitability when making decisions about inventory, logistics, storage, and handling of products.
- **Vendor Managed Inventory System (VMI):** Vendor managed inventory is a coordinated economics strategy in which vendors are empowered to manage the inventory of the buyer.
- **Integrated inventory models:** An integrated inventory model, which is crucial for making decisions to maximize profitability, integrates perspectives from producers, vendors, and consumers.

1.3 Methodologies to derive optimal solution in inventory system

1.3.1 Intermediate Value Theorem

If f a continuous function is on $[\alpha, \beta]$ and $f(\alpha) \neq f(\beta)$, If L is a some number lies between $f(\alpha)$ and $f(\beta)$ then there must be at least $\gamma \in (\alpha, \beta)$ for which $f(\gamma) = L$.

1.3.2 Optimization techniques

Optimization is the technique of achieving the optimal possible result given certain constraints. Several managerial and technological decisions must be made at various phases during the manufacturing, building, structure, and ongoing operations of anything. The final objective of all of these decisions is to maximize (profit) or minimize (cost). The method of maximization or minimization is called the optimization problem of a mathematical function of one or more variables. The function is known as an objective function. The optimization problem is solved with some limitations or constraints. Single

objective non-linear programming problem and Multi-objective non-linear programming problem are main two types of optimization problem.

Single objective non-linear programming problem: The optimization problem have a single objective function is called single objective mathematical programming problem. The minimizing of this type of problem can be expressed as:

$$\begin{aligned} &\text{Determine } z = (z_1, z_2, \dots, z_n)^T \\ &\text{which minimize } f(z) \\ &\text{subject to } z \in X \\ &\text{where, } X = \{z : g_j(z) \leq 0, j = 1, 2, \dots, m; z_j \geq 0, i = 1, 2, \dots, n\} \end{aligned} \quad (1.1)$$

where, $f(z)$ and $g_j(z)$, $j = 1, 2, \dots, m$ are defined on n^{th} -dimensional set. The objective functions and the constraints are linear in single objective mathematical programming problem, it become single objective linear programming problem (LPP).

An optimum solution to the problem is z^* which fulfilled all the constraint. The problem defined in (1.1) is to identify an optimum solution z^* such that for each z , $f(z) \leq f(z^*)$ for maximization problem, and $f(z) \geq f(z^*)$ for minimization problem and z^* is optimal solution.

Multi-objective non-linear programming problem: Multiple variables cause the problem to become more complicated. Decision-makers find it essential to evaluate the best possible solutions in cases when there are many objectives, taking into account a variety of criteria. The form of multi-objective non linear programming problem is:

$$\begin{aligned} &\text{Determine } z = (z_1, z_2, \dots, z_n)^T \\ &\text{which minimize } F(z) = (f_1(z), f_2(z), \dots, f_k(z))^T \\ &\text{subject to } z \in X \\ &\text{where } X = \{z : g_j(z) \leq 0, j = 1, 2, \dots, m; z_j \geq 0, i = 1, 2, \dots, n\} \end{aligned} \quad (1.2)$$

where, $f_1(z), f_2(z), \dots, f_k(z)$ ($k \geq 2$) are objectives. Here observed that, if the objectives of the problem are minimize $f_l(z)$, for $l = 1, 2, \dots, k_0$ and maximize $f_{k_0+l}(z)$, for $l = k_0 + 1, k_0 + 2, \dots, k$, then the mathematical structure of objective function is:

Minimize $F(z) = (f_1(z), f_2(z), \dots, f_{k_0}(z), -f_{k_0+1}(z), -f_{k_0+2}(z), \dots, f_k(z))^T$, the constraints are same as (1.2). The linear functions $f_l(z), (l=1, 2, 3, \dots, k)$, and $g_j(z), (j=1, 2, \dots, m)$, the corresponding problem is multi objective linear programming problem. When all or any one of the above functions is non-linear, it is refereed as multi-objective non-linear programming problem.

1.3.3 Local minima

If $z^* \in X$ is said to be a local minima of (1.1) if there exists $\varepsilon > 0$ such that $f(z) \geq f(z^*)$, $\forall z \in X : \|z - z^*\| < \varepsilon$.

1.3.4 Convex function

If the Hessian matrix $H(z_1, z_2, \dots, z_n) = \left[\frac{\partial^2 f}{\partial z_i \partial z_j} \right]_{n \times n}$ is semi-definite/positive definite then a function $f(z_1, z_2, \dots, z_n)$ is said to be convex.

1.3.5 Global minima

If $f(z) \geq f(z^*), \forall z \in X$ then $z^* \in X$ is said to be a global minimum of (1.1). Otherwise if the function $f(z)$ is convex then the local minimum solution becomes global minimum.

1.3.6 Convex programming problem

If $f(z_1, z_2, \dots, z_n)$ and the constraints $g_j(z_1, z_2, \dots, z_n), j=1, 2, \dots, m$ are convex then the problem defined in (1.1) is said to be convex programming problem.

1.3.7 Analytic method for single-objective problem

Necessary condition for optimality: If a function $f(z)$ is defined for all $z \in X$ and has a relative minimum at $z = z^*$, where $z^* \in X$ and all the partial derivatives $\frac{\partial f(z)}{\partial z_p}$ for $p = 1, 2, \dots, n$ are exists at $z = z^*$ then $\frac{\partial f(z)}{\partial z_p} = 0$.

Sufficient condition for optimality: The sufficient condition for a stationary point z^* to be an extreme point as the hessian matrix of $f(z)$ evaluated at $z = z^*$ then,

1. A point z^* a minimum point if hessian matrix positive definite, and
2. A point z^* a relative maximum point if hessian matrix negative definite.

1.4 Sensitivity analysis of parameters

All models in this research study are verified using numerical examples and sensitivity analyses. Sensitivity analysis determines the sensitivity of parameters and examines the model's accountability. Using mathematical software like Maple 18, MATLAB, or Mathematica, sensitivity analysis is done to determine how different parameters affect the optimal solution of the suggested inventory model by changing each parameter individually from -20% to 20% or from -40% to 40% while leaving the others unaffected.

1.5 Layout of thesis

In this proposed thesis, three scenarios of inventory-related problems are addressed and answers are provided. The suggested thesis is divided into five parts and ten chapters:

- Part I: Introduction and literatures review.
- Part II: The inventory models of the "new and buyback used products" concept.
(Inventory model of scenario 1)
- Part III: The inventory models with carbon emissions and green investments.
(Inventory model of scenario 2)
- Part IV: The inventory models that incorporated freshness and greening efforts for perishable products. (Inventory models of scenario 3)
- Part V: Summary

Part I

(Introduction and literatures review)

This part I contains two chapters: chapter 1 and chapter 2.

Chapter 1: Introduction

The basic ideas, terminologies, optimal solution methodologies, included in first chapter are those which are most commonly used in this field of research. Layout of thesis work mentions in this chapter.

Chapter 2: Literature Review

The relevant literature for the planned research is included in Chapter 2. The objectives of proposed research, research gap and original contributions mentioned in this chapter.

Part II

(Scenario 1: The inventory models of the "new and buyback used products" concept)

This part II divides into two chapters: chapter 3 and chapter 4.

Chapter 3: Retailer's optimal inventory decisions for new products and a buyback decision for used products

This chapter outlines the optimal replenishment and pricing strategies for an inventory system from the retailer's point of view and focuses on inventory policies for non-deteriorating and deteriorating products in which a retailer sells new products as well as collects and sells used products to customers. The retailer satisfies market demand by selling new products as well as buying used products back from customers, and shortages are not permissible. The demand is sensitive to selling price and exponentially decreases with time for new products and linear demand for used products. We discussed two models as below and analyse the deterioration effects on retailer's profit, and both are validated by numerical examples, sensitivity analysis and using sensitivity analysis the managerial insights are derived.

Model 3.1 Optimal inventory decision for non-deteriorating products

Model 3.2 Optimal inventory decision for deteriorating products

Chapter 4: Optimal pricing and replenishment strategies for new products and a buyback strategy of used products from the retailer's points under partial backlog shortages

This chapter posits the assumption that the retailer engages in the trading of new products, as well as the practice of buying back used products from the customers and subsequently reselling them. Two methods of meeting consumers demand are through the introduction of new products and the repurchase of used products. However, there remains backlog of partially unsatisfied demand. The evaluation of optimal inventory policies is conducted for both non-deterioration and deterioration products, with the objective of maximizing the total profit of retailer. The impact of shortages periods, backlogging rates, and price discounts on used buyback products, and deterioration effects on decisions, are determined. The validation of optimal solutions is achieved through the utilization of a numerical example, while the concavity of the objective function is illustrated both graphically and numerically. The chapter discusses the performance of a sensitivity analysis in order to ascertain the impact of various parameters on optimal outcomes. Additionally the chapter provides a comprehensive overview of the managerial insights derived from the analysis. Finally, the chapter concludes by summarizing the key finding and implications. This chapter classified into two models as below:

- Model 4.1 Optimal inventory strategy for non-deteriorating products for which shortages are partially backlogged
- Model 4.2 Optimal inventory strategy for deteriorating products for which shortages are partially backlogged

Part III

(Scenario 2: The inventory models with carbon emissions and green investments)

This part III includes three chapters: chapter 5, chapter 6 and chapter 7.

Chapter 5: An EOQ model for deteriorating products with green technology investments and a trade credit payment system

In this chapter, we developed an EOQ model with six cases in the form of carbon regulation policies and trade credit payment strategy for perishable products whose deterioration depends on expiration dates. The consumer demand depends sustainability credentials and products price. The demand is a function of carbon reduction (as a green investments), and the selling price. Investments in green technology that help reduce carbon emissions are taken into account. Optimized the retailer's total profit by considering carbon tax and carbon cap-trade policies with and without trade credit payment systems with respect to the optimum value of green investment cost, selling price, and

cycle time. The analysis of the different cases suggests that a carbon cap-and-trade policy with trade credit financing is better than a carbon tax policy and will yield the highest profit. Using the hessian matrix method and graphically, verified the optimality of objective function. Numerical examples are used to demonstrate the solution process, and sensitivity analysis is used to explore strategically possibilities.

Chapter 6: Sustainable economic production quantity (SEPQ) model for inventory having green technology investments-price sensitive demand with expiration dates

In this chapter, we proposed manufacturer's sustainable economic production quantity model with carbon cap-tax mechanism for perishable products whose deterioration depends on expiration dates and demand depends upon the green investments and selling price. Setting up the production system, production process, storing process, product deterioration, and environmental impacts are the sources of carbon emissions taken into account. Obtained the manufacturer's optimal policy with considering green investments. A numerical example has been looked at to demonstrate the accountability of the model. Sensitivity analysis has highlighted the management implications of the feasible solution with regard to parameters. There are also a few closing remarks and potential future applications offered.

Chapter 7: Optimal green investments and replenishment decisions in vendor-managed inventory systems for non-instantaneous deteriorating products with partial backordering

The current chapter deals with the individual green supply chain model and the vendor-controlled green supply chain model, in which products are non-instantaneously deteriorating. The demand depends on green investments (carbon reduction function) and their promotional levels during stock available period and shortages are admissible, it is partially backordered from buyer side. It is assumed that demand remains constant without the effect of green investments and promotion levels during the stock-out period. Evaluated the green investment helps to reduce the carbon emission cost. The study shows that the VMI model is better than the traditional model from a green investments, carbon emission reduction, and total cost point of view. Derived the impacts of the shortage period, the distance between supplier and retailer, fuel utilization, product deterioration, and carbon emissions. Our objective of proposed study is to optimize the total cost of

supply chain with respect to cycle time and green investment cost. For the proposed chapter model's authentication, numerical examples and sensitivity analyses are provided.

Part IV

(Scenario 3: The inventory models that incorporated freshness and greening efforts for perishable products)

This part IV contains two chapters: chapter 8, chapter 9.

Chapter 8: Optimal greening efforts, pricing and inventory strategies for non instantaneous deteriorating perishable products under price, freshness and green efforts dependent demand with price discount

The EOQ model for non-instantaneous deteriorating products with freshness, selling price, and greening efforts demand is formulated in this chapter. Demand for perishable products were determined not only by freshness and price, but also by the consumer's preference for greenness. Perishable products' physical deterioration and freshness-based deterioration are considered. The quality of the product decreases during the deterioration period, and hence the retailer offers a price discount to boost demand. An ordering and pricing strategy is formulated with concern for product greenness and freshness. The optimality of objective function is validated though theoretically and graphically. For the authenticate proposed model, real and numerical examples are taken. For the effect of parameters optimal decision sensitivity analysis is derived. The strategic usefulness of the model is mentioned as a managerial insight, finally concluding the chapter with the future scope of the study.

Chapter 9: An EPQ Model for Delay Deteriorating Perishable Products with Price, Freshness and Greening Efforts Dependent Demand under Markdown Strategy

Entrepreneurs implement different policies in their organizations to advance business. A markdown policy is offered by the producer in order to increase the sales of inventory and enhance the profit from clearing stocks at the end of the product's life. In our chapter, we optimized the production quantity, quantity under markdown policy offer, production time, markdown offering time, total cycle time, and markdown percentage such that the total profit of the producer is maximized. The level of freshness, greening of perishable products and price can be regarded as the main factors influencing a buyer's purchase behaviour. Demand is depending on price and green initiatives at the start of the inventory

cycle. After production stops, products that are affected by physical as well as freshness-based deterioration. During this time, demand patterns depend on freshness, price, and greening efforts. For the purpose of model justification, the problem has been represented in a mathematical model, and a solution procedure and example have been provided. We employ sensitivity analysis to highlight the analytical findings and provide major management implications as a conclusion.

Part V Summary

Chapter 10: Conclusion and future research scope of the study

A summary of the thesis, its constraints, and the range of additional research have been provided at the end of this thesis. The bibliography is included at the end of this chapter.

CHAPTER-2

Literature Review

2.1 Introduction

This chapter incorporated the literature survey to the proposed study on the inventory modelling. The study of literatures is distributed in four sections: (i) a literature review on inventory models regarding product deterioration and various demand patterns with different factors likes, non-instantaneous deteriorations, shortages, trade credit policy, models on product expiration dates, VMI system, etc., (ii) a literature that focuses on scenario 1 models, i.e., inventory models for new and buyback used products and reverse logistics; (iii) a literature review which is related to scenario 2 models, i.e., models on carbon emissions and green investment policy; and (iv) a literature review on scenario 3 models, i.e., the literatures on product freshness, greening efforts, and markdown policy. Finally, the research gaps and objectives of purposed research work are mentioned in this chapter.

2.2 Inventory modelling on deterioration and various demand patterns

Economic order quantity (EOQ) or economic production quantity (EPQ) are the earliest and most widely used approaches for creating an inventory system to reduce overall cost while maintaining the optimal value of replenishment quantity or production quantity and cycle time. Harris[2] and Taft[3] introduced the EOQ model and EPQ model, respectively, which are focused on inventory modelling. In their models the demand pattern is constant, the replenishment rate is instantaneous, and shortages are avoided. These are three basic assumptions, and ordering cost, procurement cost, and holding cost are taken into account to develop an inventory model. For the derive economic order quantity, the well know

formula is $Q^* = \sqrt{\frac{2AD}{h}}$, where A is ordering cost or set up cost per order, D is constant

demand rate per year, h is a carrying cost per unit per year, this formula for Q^* is also known as the *Wilson* or *Harris* lot size formula.

Product's deterioration: Numerous researchers have occasionally conducted extensive studies on concerns with inventory modelling for deteriorating products. A process that prohibits an item from being used for its intended purpose is referred to as ***Deterioration***. Examples of such processes include decay, physical degradation, decompose, vaporization, loss of effectiveness, etc. A product's rate of deterioration expresses both its physical condition and its expected lifespan. Different types of inventory models' deterioration behavior include those with a fixed product lifetime, those with an age-dependent deterioration rate, or a probabilistically distributed life, and those with a time-, stock-, or constant deterioration rate. Whitin[4] who took consideration of fashion products deteriorating after a certain period of storage time, was the first to explain deterioration. The first framework for a negative exponentially deteriorating inventory was developed by Ghare and Schrader[5]. They noticed that the value of some products decreased over time proportionally to a negative exponential function of time. By highlighting the possibility of initiatives to reduce expenses and advancements in supply replenishing policies, they highlighted the significance of taking the impact of deterioration into account in inventory modelling. This analysis resulted in the formulation of a model of inventory management with deterioration by the first-order differential equation,

$$\frac{dI(t)}{dt} = -I(t)\theta - f(t) \quad (2.1)$$

Where, $I(t)$ is the stock level at time t , θ represents constant rate of deterioration of product and $f(t)$ is the rate of demand at time t .

Covert and Philip[6] extended the model of Ghare and Schrader[5] by using Weibull distribution and Tadikamalla[7] extended the model of Ghare and Schrader[5] with considering gamma distribution. An up-to-date review on inventory systems for deteriorating items is presented by Raafat[8], Shah and Shah[9], Li et al.[10], Goyal and Giri[11], Bakker et al.[12], Janssen et al.[13] etc. State of the art literature study for perishable products provided by Chaudhary et al.[14]. The authors Wu et al.[15], Musa and Sani[16], Ouyang et al.[17], Maihami and Karimi[18], etc. were taken constant rate of deterioration in their study of inventory modelling.

Various demand patterns: Demand plays a significant impact in the market for inventory management. The demand rate is supposed to be constant in the traditional EOQ model,

which is generally inaccurate. Therefore, more emphasis has been dedicated to studies on inventory management with demand fluctuations. Demand distributions as deterministic or stochastic[19]. The deterministic demand can be classified as constant/uniform, price-dependent, time-varying, stock-dependent, combination of time, price, stock level, green investment dependent, credit period dependent, promotional level, etc. and stochastic demand can be categorised with known and arbitrary demand distribution. The market demand for most of the products may depend on time, price of the products, and stock, but as per a recent study, the demand may also depend on product quality, product age, greening level of the products, etc. The constant demand in the inventory system taken by Covert and Philip[6], Cheng et al.[20], Dye and Hsieh[21], Sarkar et al.[22], Toptal et al.[23], Mishra et al.[24] etc. The time dependent demand may be linear form i.e. $a+bt$, where $a \geq 0$ scale demand, $b > 0$ if demand increases with time, $b < 0$ demand reduces with time or quadratic form i.e. $a+bt+ct^2$, where $a \geq 0$ scale demand, if b and c both non zero and both positive then demand increases and if b and c both non zero and both negative then demand decreases or exponential form i.e. ae^{bt} , where where $a \geq 0$ scale demand, $b > 0$ indicates demand is high with time and $b < 0$ demand is low with time by Xu and Wang[25], Goyal and Giri [11]. The demand for a product may rise in some real-world scenarios if the price per unit is reduced. Linear form $a-bp$, where $a > 0, b > 0$, exponential form ae^{-bp} where $a > 0, b > 0$, logit form $\frac{a}{1+e^{bp}}$, where $a > 0, b > 0$ and logarithmic $a-b \ln p$, where $a > 0, b > 0$ etc. price sensitive demand patterns discussed by Avinadav et al.[26], Goyal and Giri [11]. Mandal and Phaujdar[27], Shah et al.[28] and Baker and Urban[29] discussed the inventory models with linear and nonlinear stock dependent demand respectively. The detailed literatures on demand in inventory modelling that are most suitable to our suggested study, including price-time dependent demand, green investments and its promotion-related demand and product freshness-sensitive demand, is provided in the next respective sections.

2.2.1 Inventory models on price and time dependent demand and deterioration

The price of a product is a priority for today's budget-conscious customers. Studies on the most effective pricing methods are currently getting a lot of attention because the selling price of a product is a crucial factor in determining demand. A product's selling price and

demand frequently appear to be inversely proportional. The retailer's ordering quantity is affected by demand, and demand is dependent on product selling price. It is shown that retail price and lot size are correlated. Inventory system with price relevant demand was first formulated by Whittin [30]. Optimal pricing and replenishment quantity policy for deteriorating products were evaluated by Kim et al.[31]. Wee[32] introduced inventory policies for a deteriorating products with price elastic demand rate that reduces with time. The demand rate in the study of Wee [32] is the form of $(a - bp)e^{-\epsilon t}$, where $a, b, \epsilon > 0$ and p is the selling price of product. The dynamic pricing and lot sizing inventory policies for perishable products was developed by Abad[33]. Price dependent demand for ameliorating items taken by Mondal et al.[34]. Mukhopadhyay et al.[35] developed the joint pricing and ordering mechanism for deteriorating inventory. The demand rate is non-negative continuous function decline with selling price was considered by Chang et al. [36] and Dye[37]. The inventory policies with price related demand, stochastic lead time with advance payment system was developed by Maiti et al.[38]. Liu et al.[39] obtained the joint pricing policy for perishable foods with price-quality based demand. Thereafter, the numbers of authors, Shah and Vaghela[40], Sundararajan et al.[41], Dey et al.[42], Shukla and Suthar[43], Shaikh and Cardenas-Barron [44], Giri and Masanta[45], Shah and Naik [46] considered price-dependent demand in their research. Recently, Liu et al.[47] developed the pricing policy for overconfident customers in dual supply chain. Price dependent stochastic demand in newsvendor problem taken by Khan et al.[48].

Numerous inventory items like electronic products, clothes, seasonal products, etc. might not always be appropriate under the concept of a constant demand rate. In the development stage of a product's life cycle, many products demand increases with time. The launch of the latest features in products may cause customer choices to shift, resulting in a decrease in demand for some products over time. Time-dependent demand received a lot of attention among them. Goyal and Giri [11] discussed about continuous-time and discrete-time varying demand. In the development of inventory modelling, generally continuous-time varying demand patterns considered it may be linearly increasing/decreasing, i.e. $D(t) = a + bt, a > 0, b > 0$ or exponentially increasing/decreasing, i.e. $D(t) = ae^{bt}, a > 0, b > 0$. Beginning in the nineties century, Xu and Wang [49] obtained the inventory policy for deteriorating products with time dependent demand. Linearly time varying demand considered by Xu and Wang[49] for the exponentially deteriorating products. Exponentially decreasing demand over time for deteriorating products was first

taken by Hollier and Mak[50] . Hariga and Benkherouf [51] extended the study of Hollier and Mak [50] including exponential increasing and exponential decreasing demand over time. Chung and Ting[52] developed first heuristic model for deteriorating products with time dependent linear demand. The demand rate, deterioration rate and all cost components influences with time assumed in the study of Giri and Chaudhuri[53].Teng[54] constructed the inventory policy for deteriorating products with time varying demand. A time dependent trapezoidal demand was taken into account by Cheng and Wang[55]. Maihami and Nakhai Kamalabadi[56] studied the inventory model for non-instantaneous deteriorating products with time and price dependent demand. Price- and ramp-type time proportional demand for seasonal deteriorating products was taken by Wang and Huang[57]. Shah and Vaghela[58] developed EPQ model with price and time dependent demand under trade credit policy. Shah et al.[59] considered the stock dependent demand and time dependent fixed life of products in inventory modelling. Adak and Mahapatra [60] designed the multi items deteriorating inventory system with time dependent demand. Shah et al.[61] derived the optimal inventory strategies for price and time based demand including advertisement.

2.2.2 Inventory models on non-instantaneous deterioration

Products like radioactive substances, flammable liquids, fashionable products, drugs, high tech products, and smart phones have a limited amount of time to maintain their quality or original conditions. Such a circumstance is sometimes referred to as ***non-instantaneous deterioration***. Non instantaneous deterioration means, the deterioration starts after a certain time period by Wu et al.[15]. Recently manufactured or produced products do not degrade immediately after manufacturing, production, or storage; instead, they gradually deteriorate over time. Non-instantaneous deterioration of products concept first time taken by Wu et al.[15] and Ouyang[17] in inventory modelling. Also, they came to the conclusion that the cost of an inventory system could be kept to a minimum if retailers could lessen the impact of deterioration by providing suitable facilities at storage areas. Many authors Dye[62], Soni and Patel[63] , Maihami and Karimi[18], Shah et al.[64], Jaggi et al.[65], Geetha and Uthayakumar[66], Rabbani et al.[67], Wang et al.[68], Mashud et al. [69] incorporated non instantaneous deterioration in their study. Sundararajan et al. [70] discussed an EOQ model of non-instantaneous deteriorating items with price, time-dependent demand and exponential backlogging rate.

2.2.3 Inventory models on product's expiration dates

The word "*expiry date*" refers to the point at which a product has reached the end of its useful life or has become outdated. Products like photographic film, medications, packaged foods, electronics items, and perishable products exhibit this behaviour. These kinds of products degrade with time and may lose value at the expiration date slowly or rapidly. All perishable products not only deteriorate with time, but they also expire with time by Fujiwara and Perera[71]. Hsu et al.[72] developed the lot sizing model for deteriorating products which have expiration date. Sarkar[73] developed EOQ model with trade credit policy for the perishable deteriorating product with expiration date. Freshness and fixed shelf life of product discussed by Herbon[74]. In the study of Chen et al.[75], expiration dates frequently play a significant role in customer decision-making. Wu et al.[76] obtained the inventory strategies with different payment system for the perishable products. Shah et al.[77] identified the inventory policies for the product have a maximum fixed life with quadratic demand. Deteriorating products with maximum lifetime considered in supply chain model by Pramanik et al.[78]. Product's expiration duration, price sensitive demand and preservation technology concept taken by Gautam et al.[79]. Shi et al.[80] obtained the optimal policies with different payment systems and carbon tax regulations for perishable products. Kamaruzaman and Omar[81] developed the inventory policy for the perishable product under freshness and price sensitive demand. More recently, Yadav and Khanna[82] developed the sustainable inventory model with carbon tax for the perishable product. Sepehri et al.[83] constructed sustainable inventory model for deteriorating products with maximum lifespan and delay in payment.

2.2.4 Inventory models with shortages

The fundamental models of inventory system where shortages are avoided but many researchers included shortages in their study, if shortages are allowed, replenishment cycle time increases and the carrying cost of the inventory is reduces. If the cost of the inventory is high per unit, then backorders due to shortages are beneficial. The backlogging rate say $S(t)$, is anticipated to be variable and it is depends on the waiting duration of next delivery of stock say $(T - t)$, where T is duration of ordering cycle. The portion $1 - S(t)$ is lost sales. If duration of waiting is increases, $S(t)$ is decreases. Therefore, the percentage of consumers who like awaiting a subsequent replenishment declines over time t . According

to the literature[33] relevant to this, the backlogging rate in rational form is $S(t) = \frac{S_0}{1 + \delta(T-t)}$ or exponential form of backlogging rate is $S(t) = S_0 e^{-\delta(T-t)}$, where S_0 is backordering intensity and δ is backlogging parameter by Abad[33]. As per Abad[33], the shortages are partially backlogging if $0 < \delta < 1$, In this circumstance, some consumers are prepared for wait their demand till next delivery of stock. $\delta \rightarrow 0$ gives the complete backlogging, in this situation all consumer will wait for next delivery of products. If $\delta \rightarrow \infty$, means completely lost sale occurred and all consumers moves to other places for their demand. The rational form and exponential form of backlogging rate studied by Abad[33]. Lot sizing policy for perishable product with finite production rate and partial backlogging developed by Abad[84]. Then after Abad[85] investigated the pricing and ordering policies for reseller under shortages which are partially backlogged. The exponential form of backlogging rate is taken by Dye et al.[37], Yang, Teng and Chern[86], Maihami and Kamalabadi[56] . Jani et al.[87] also taken the exponential form of backlogging rate and optimize the preservation investment with trade credit policy.in their study. Debata and Acharya[88], taken rational form of backlogging rate with time dependent weibull deterioration rate. Chakraborty[89] developed the model for backlogging rate is exponentially decreasing function and time dependent weibull deterioration rate. Chang et al.[36], Shah and Shukla[90], Sarkar and Sarkar[91], Shukla and Suthar[92] etc. considered the rational form of backlogging rate. Many authors like Palanivel and Uthayakumar[93], Shah et al.[94], Duary et al.[95], Khan et al.[96]etc. taken the partially backordering shortages in their study.

2.2.5 Inventory models on trade credit payment system

In today's business scenario, suppliers permit retailers to postpone payments for a certain period of time in order to increase demand and attract additional consumers. This delay in payment offered by the supplier to retailer is known as a credit period. This economic strategy is termed as a trade credit policy. As a result of this approach, the supplier or retailer will have the chance to keep the products on the market for a longer period, increase sales, and gain more profit. Regarding to this point, Haley and Higgins[97] investigated the connection between inventory policy and trade credit financing. In the study of Chapman et al.[98] included credit payment permissible by supplier in EOQ model. Goyal[99] proposed an inventory model to optimize the ordering quantity under the

assumption that the supplier gives the retailer a certain amount of time to settle the account. In the same year, Dave[100] extended the model of Goyal [99] by considering that the product selling price is more than its purchasing value. Deterioration of product included by Aggarwal and Jaggi[101] in inventory model with trade credit policy. Hwang and Shinn[102] obtained the pricing and ordering policies for the retailer under allowable postponed in payments. Jamal et al.[103], Chang and Dye[104] developed the model for deteriorating products under partial backordering and trade credit policy. Huang[105] extends Goyal[99] study in which trade credit policy apply to supplier to retailer and retailer to customers. The credit period given by the supplier to the retailer is longer than the retailer gives to customers. Chang[106] constructed model with deteriorating products in which supplier credits correlated to replenishment quantity under inflation. Shah and Trivedi[107] investigated inventory policies for deteriorating products in which random supply and supplier permit to delay in payment. Yang and Wee[108] designed a collaborative supply chain model including trade credit payment policy and negotiation scheme. Liao[109] developed EOQ model with two level credit period and exponentially deteriorating products. Optimal replenishment decision in EPQ model with trade credit policy and shortages obtained by Hu and Liu[110]. Soni et al.[111] provided an enrich review on inventory system and trade credit. Chen and Teng[112] constructed an EOQ model with the deterioration rate linked to expiration date and trade credit payment policy. Teng[113], Sharmila and Uthayakumar[114] , Tiwari et al.[115], Pramanik[78], Shah and Jani[116], Sarkar et al.[117], Shi et al.[80], Shah et al.[59],etc.; authors are taken the trade credit policy with various assumptions in their research. Molamohamadi[118] ,Kawale-Sanas[119] given the up to date literatures review on trade credit policy in inventory modelling. Qin et al.[120] developed sustainable inventory model for trade credit and ordering policies under carbon tax and cap-trade regulations. Green credit and trade credit financing with carbon emissions in supply chain was discussed by An simin et al.[121]. Mahato et al.[122] derived the sustainable ordering decisions with carbon emission, trade credit and partial backordering.

2.2.6 Inventory models on VMI policy

Numerous businesses and industries link with one another and build collaborations in the supply chain under the supervision of a certain framework, such as a vendor-managed inventory strategy. The collaborations increases supply chain transparency, which leads to

cost reductions. In this area, Narayan and Raman[123] studied the effect of VMI on service level and supply chain profit. Achabal et al.[124] developed the inventory policies with VMI. Yao et al.[125] examined that, in VMI system given the lower inventory costs for supply chain members and enhanced customer service levels, including reduces order cycle times and higher replenishment quantities. Darwish and Odah[126] formulated the VMI models with single vendor and multiple buyers by under various assumptions. Shah et al.[127] developed model on single vendor and more than one buyers supply chain system with demand is quadratic form. Yu et al.[128] design the vendor managed inventory system for deteriorating raw material and finished products, and minimizing the cost function of supply chain with shortages. Setak and Daneshfar[129] described the model for single supplier-single retailer with stock dependent demand and constant deterioration and shortages, to evaluate the results for VMI and traditional supply chains. Taleizadeh et al.[130] formulated joint pricing, ordering and production policy for VMI system. Tat et al.[131] designed a model in which Vendor managed inventory (VMI) system is the type of inventory system such that the vendor obtained the data regarding the status of stocks from buyer, thus vendor is responsible for deciding ordering policy and replenish time of system as per the knowledge sharing between both. Green vendor managed inventory supply chain model formulated by Jiang[132] with carbon trading regulations. Using metaheuristic algorithms, Rabbani et al.[133] formulated VMI model for deteriorating inventory. Soni et al.[134] developed VMI model and traditional inventory model for non-instantaneous deteriorating product, shortages are allowed with partial backlogging in which identified that VMI model is better than traditional model. Bai et al.[135] examined effects of carbon emission reduction on supply chain coordination with vendor managed deteriorating product inventory. Hsiao[136] developed VMI policies with carbon emissions. Hariga et. al.[137] developed single buyer multi retailer VMI system under carbon cap regulation.

2.3 Inventory modelling on new and buyback used products and reverse logistics

Buyers are not only looking at purchasing newly manufactured or recently launched goods, but they are also willing to buy a used product from a seller due to environmental concerns and because it has the same characteristics as a new product and is less expensive. On the other side, the main concern right now is to improve environmental support strategies in

order to protect our finite natural resources and lower waste generation as required by law. Customers choose to buy products from manufacturers with a green reputation because they are concerned about environmental issues. Because of this, numerous businesses have started gathering used products that customers throw away. The retailer resells the products to a new consumer at a discounted price after refurbishing or recycling those items, so the retailer earn revenue from new products as well as take back used products by Chen et al[138]. As early as the 1960s, the idea of considering reusable products as a different source of supplies was investigated. An inventory policy for repairable products with constant demand was first presented by Schrady[139], who assumed a fixed lead time for outside delivery of products and a recovery rate. Schrady's[139] model was expanded by Nahmiasj and Rivera[140] to support a finite repair cost under the presumption of limited storage space in the repair and production facilities. Mabini et al.[141] also extended Schrady's[139] model for multi-items having the same repair facility. Richter[142] first optimized waste disposal rate with variable set up number of production. Richter[143] obtained the importance of cost factors of repairing system in EOQ model. Reverse logistics includes the gathering and sale of old products. The first step in reverse logistics is to gather or buy used products from buyers. Carter and Ellram[144] investigated that reverse logistics is an operations of refurbishing, reusing, and minimizing the quantity of raw-materials to become more sustainable efficient. Many more researchers developed their study on reverse logistics, recycling, and reuse of products. Richter and Dobos[145] developed EOQ repair and waste discarding problem and demonstrated that either the total trash disposal or the total repairing technique is the dominant one. Koh[146] formulated the optimal ordering policy of new products and buyback policy for used products simultaneously, the market demand is satisfied by the new products and used products which take back from consumers. Electronics products waste recycling and again using repaired the same products is discussed by Daniel et al.[147]. Richter and Dobos[148] developed model on production inventory control with reverses logistics system. Savaskan et al.[149] formulated closed loop supply chain for remanufacturing. Nagurney and Toyasaki[150] developed the inventory system of batteries recycling and reusing. Heese et al.[151] investigated the advantages of buyback strategy in competitive market. Gonzlez-Torre and Adenso-Daz[152] investigated the result from glass recycling. Pati et al.[153] designed an inventory model on paper recycling. Pokharel and Mutha[154] observed the ongoing advancements in both research and practical applications with in field of reverse logistics. Kannan et al.[155] developed a closed loop supply chain model for battery

recycling utilizing a genetic algorithm approach. Alfonso-Lizarazo et al.[156] presented modelling on reverse logistics operations in the agro-industrial field. Jaber et al.[157] studied ordering policies for imperfect product including buy and repair option. Govindan et al.[158] conducted a thorough examination of reverse logistics and closed-loop supply chain, offering a comprehensive review of the topic. Chen et al.[138] formulated EOQ model for retailer centric decisions, in which retailer sells the new product and buyback same product for resale. Optimized the replenishment quantity of new products and buyback quantity of used product, demand satisfied by new and used products in Chen et al.[138]. Shah and Vaghela[40] formulated inventory policies for new products and take back policy for used products. Singh and Rana [159] developed optimal refilling policy for new deteriorating products and take back policy for old deteriorating product under inflation. The demand of new products is a quadratic function of time is taken by Shah and Vaghela[40], Singh and Rana[159]. New and refurbished products deteriorating green supply chain model formulated by Rani et al.[160].

2.4 Inventory modelling based on carbon emissions and green investments

In the past few decades, two of the most severe concerns affecting the atmosphere have been global warming and environmental degradation. Carbon emissions are becoming an issue for both humanity and the natural world because of the hazards of changing the climate and resulting global warming. Governments now give top importance to reducing carbon emissions, which are a major cause of global warming. The approval of the Kyoto Protocol in December 1997 was a significant step in reducing carbon emissions and mitigating their effects on climate change[161]. The World Bank (2015) proposed an addition carbon pricing techniques to help regulate the carbon emissions produced by various sources [162]. "Sustainable development" refers to a process that takes into account not only ecological considerations but also economic and social factors as well. The fundamental focus of sustainable inventory management is to achieve a trade-off between environmental concerns and profitability by Kleindorfer[163]. Global warming is mostly caused by carbon dioxide emissions from commercial and commercial operations, according to the Inter-governmental Panel on Climate Change (IPCC)[164]. Inventory management represents one of the areas concerned with carbon emissions and environmental sustainability. Emissions trading, also known as cap-and-trade, is one of the

Kyoto Protocol's most effective strategies, supported by the United Nations[161]. A cap-and-trade regulation specifies limits on an industry's carbon emissions and governs the exchange of emission allowances among different businesses. Many nations, areas, and local governments around the world implement a carbon tax or another type of energy tax. As part of its carbon tax, the government charges an amount for each unit of carbon emissions produced by business sectors. In addition to carbon policies, investing in various technologies that reduce carbon emissions also helps to save the environment. This concept, which also emphasizes the management of inventory systems, is known as green technology or sustainable technology. Green operations apply to every aspect of an inventory or production framework, including manufacturing and refurbishment, shipping, handling, and controlling waste Srivastava[165].

Carbon policies and green investments: The various carbon emission control methods, such as a carbon tax and cap-and-trade and green investments were extensively studied by several authors. Hua et al.[166] were the first to incorporate carbon emission into inventory models under cap-and-trade regulation. Bouchery et al.[167], Hovelaque and Bironneau [168] by taking into account the carbon tax policy, and Arslan and Turkay [169] by included the carbon offset policy as further extension studies of Hua et al.[166]. Bonney and Jaber [170] given an idea that vehicle emissions are a cost that becomes an additional charge in an economic order quantity (EOQ) model. The purchasing process is the source of carbon emission by Chen et al. [171] explained the model on carbon foot print in supply chain. As per Jaber et al.[172], manufacturing operations are a main source of carbon emissions, and using reduction technology, carbon emissions can be reduced. By considering price dependent demand in the work of Jaber et al.[172] extended by Krass et al.[173] and Lou et al.[174]. Huang and Rust[175] used the green investment in inventory model. Alzaman[176] designed the detail literature review on green supply chain modelling. Toptal et al.[23] discussed about the green technology investment under carbon cap and trade and carbon tax mechanism. Toptal et al. [23] considering that the ordering, persevering, and procuring inventory are the causes of emissions simultaneously. Hu and Zhou[177] derived the joint optimal decisions of pertaining to pricing and carbon emission reduction. Qin et al.[120] conducted an assessment of a trade-credit inventory model in the context of a carbon tax, and carbon cap and trade policy, with a specific focus on the impact of demand variability based on the credit period. Lin and Sarker[178] developed a mathematical model to analyze inventory management of imperfect quality items,taking into account the implementation of a carbon tax policy. In their study, Lou et al.[174]

derived the optimal policies for sustainable inventory models aimed at maximizing profits through the implementation of green technology investments. These strategies were specifically designed to reduce emissions in accordance with the carbon tax and carbon cap-trade policy. In their study, Tiwari et al.[179] investigated viable framework for managing perishable and imperfect goods with focus on sustainability. The impact of green technology investment on the green manufacturing process was assessed by Bhattacharyya and Sana[180]. The researcher Zand et al.[181] formulated pricing and ordering strategies that incorporate both greening level and the demand sensitive to price. Lu et al.[182] investigated the effects of carbon cap-and-trade and carbon offset policies on a perishable inventory model that incorporates carbon reduction technology to mitigate emissions within the supply chain. Shi et.al [80] developed an inventory model that incorporates a carbon tax and explores its implications under different payment system and expiration dates. Panja and Mondal [183] proposed a two-layer green supply chain model that incorporates revenue sharing contract. The deteriorating inventory model with carbon tax was formulated by Yu et al.[184]. The EOQ model under carbon tax and product expiration dates with price dependent demand was developed by Yadav and Khanna[82]. The model developed by Paul et al.[185] incorporates a demand pattern that is influenced by both price and greening level, taking into account the presence of a carbon tax regulations. Mashud et al.[186] conducted an analysis of a sustainable inventory model that incorporates the management of carbon emissions in green-warehouse farms. The inventory model for perishable products under a trade credit policy was developed by Sepehri [83] with a focus on controllable carbon emissions. Bhavani et al.[187] have proposed a sustainable green inventory system that integrates a unique eco-friendly demand model and accounts for partial backlogging in the presence of uncertainty. Daryanto et al.[188] and Mashud et al.[189] conducted a study on carbon emissions originating from transportation. Their study focused on factors such as a fuel consumption, vehicle usage, and a contribution of these factors to carbon emissions. In their study, Taleizadeh et al.[190] presented an inventory model that incorporates partially backlogged shortages. They further examined the effect of lost sales and carbon emissions in the context of trade credit payment.

EPQ models with carbon emissions: In their study, Mishra et al.[24] formulated a sustainable production inventory model that considers constant demand, green investments, and shortages. A green investment was employed to mitigate carbon emissions through the implementation of carbon cap and tax mechanism. Huang et a.[191]

formulated sustainable investment strategy that incorporates various carbon policies within a supply chain model. In their study, Shah et al.[192] investigated the EPQ model in the context of price-stock sensitive demand, taking into account carbon emissions and incorporating a carbon tax-cap mechanism that encompasses preservation and green investment. Mashud et al. [193] developed the production lot size model with green investments. The effect of multiple prepayment in green EPQ model obtained by Paul et. al [194]. Kataiya and Shukla[195] formulated manufacturer's model by considering carbon cap-tax and green investment strategy.

Green investment sensitive demand: The environmental performance increases the overall demand derived by Saadany[196]. The demand of the products depends on the consumers preferences on green investments and greening level make positive impact on purchasing choice by Zanoni et al.[197]. Ghosh and shah[198] formulated the supply chain inventory model with green sensitive consumer demand and sharing of cost concept. Aliabadi et al.[199] investigated the optimal pricing and green technology investment under fluctuating consumer demand, whereas Xia et al.[200] considered the impact of a promotion strategy. Customers' understanding of carbon emission levels, according to Tao and Xu[201], will inspire them to buy from green technology-focused firms. Maihami and Karimi[18] taken stochastic demand and promotional efforts. Hasan et al.[202] derived inventory model to optimizing inventory level and technology investment under a carbon tax, cap-and-trade and strict carbon limit regulations with carbon reduction and their promotion sensitive demand. Cardenas-Barron and Sana[203] developed two level supply chain model with promotional efforts dependent demand.

VMI and carbon emission: Jiang et al.[132] explored a green VMI supply chain model under carbon emission policy. Bai et al.[135] examined effects of carbon emission reduction on supply chain coordination with vendor managed deteriorating product inventory. A sustainable VMI system with different carbon policies was developed by Malleeswaran and Uthayakumar [204]. Sarkar and Guchhait[205] developed VMI model with green investments. Astanti et al.[206] formulated low carbon supply chain under VMI relationship and cap-and-trade policy.

2.5 Inventory modelling based on product's freshness , greening efforts and markdown policy

Perishable products, including fruits, vegetables, meat, dairy products, beverages products, etc., are currently purchased based on price and health concerns in addition to the freshness of the products. Product freshness is an essential component of its quality, and as a result, the choice of purchase for consumers also depends on the freshness of the green products.

Inventory model on product's freshness: Several researchers have hypothesized that inventory models for perishable products follow a random lifetime of products, their freshness stays the same, and their freshness does not affect demand until they have reached their expiration date. The first study to look at how demand is impacted by a product's freshness was done by Fujiwara and Parera[71]. Later, Sarker et al.[207] believed that the age of existing inventories had an unfavorable effect on demand. Tsiros and Heilman[208] examined how items lose part of their value to customers as well as some of their freshness over time. According to a study by Bai and Kendall[209], the demand for fresh produce is reliant on both its freshness and the inventory that is on display. Wang and Li[210] developed an inventory model for perishable goods with a set time frame in which quality degradation is a key factor. In an EOQ for fresh produce, where demand influences the freshness expiration date and supply levels, Chen et al.[75] examined this concept. According to a model of inventory created by Dobson et al.[211], the linear reduction in demand rate is due to product freshness declining. For a perishable product that is subject to both physical deterioration and freshness condition degradation, Agi and Soni[212] presented a deterministic model for jointly optimizing pricing and inventory control. In this model, the demand for the product is dependent on its price, stock level, and freshness level. Green fresh product cost sharing contracts considering freshness-keeping effort, model developed by Wang et al.[213]. Bhaula et al.[214] developed an inventory model for perishable non-instantaneous products for the optimize the selling price, freshness to maximize net profit under subsequent price discounts. Soni [215] constructed the inventory policies for perishable deteriorationg products under freshness based demand and price discount. To explore partial replacement policies and finite shelf lives for deteriorating goods with carbon tax in inventory model by Malakar and Sen [216]. Banerjee and Agrawal[217] formulated pricing and ordering policies for deteriorationg products with price and freshness sensitive demand under discount policy. Kaya and Bayer[218]

examined the pricing and lot sizing policies for the perishable products whose demand varies corresponding to freshness. Recently, Qiao et al.[219], Hua et al.[220], Wu et al.[221], Dharm and Lin[222], Sebatjane and Adetunji[223], Xu et al.[224], Khan et al.[225] etc. taken freshness sensitive demand in their study.

Inventory model on greening efforts: Recently, a number of initiatives have been undertaken by manufacturers and retailers to demonstrate how serious they are about going green. The activities done to assure sustainable products and reduce the impact of business activities on the environment are referred to as greening efforts. Plambeck[226] stated in a review of Walmart's green programs that "being an effective manager of the natural environment as well as being profitable are the same". Due to environmental issues, Coca-Cola, one of the biggest beverage companies in the world, has started recycling and reprocessing discarded bottles in developing nations like India through encouragement in a supply chain. Channel coordination in a supply chain with greening investment concerns was proposed by Swami and Shah[227], Li et al.[228] examined the pricing decisions for dual channel green supply chain.. Raza and Faisal[229] formulated an inventory models for joint pricing and greening effort decisions with discounts. An annual report on organic farming, food security, and environmental concerns was provided by Meemken and Qaim [230]. Between 2000 and 2015, they looked into the consequences of organic agriculture on soil quality, biodiversity, and the ecosystem. In order to account for price and stock-dependent demand rates as well as greening efforts, Shah et al.[28] developed an inventory model that utilized perishable products. Ji et al.[231] studied how green credit financing at a discounted rate motivates the supplier to improve the reduction in carbon emission levels. Shah et al.[232] developed the pricing decisions with the effect of advertisement and greening efforts for a greengrocer. Maihami et al.[233] studied the beef sector as a case study. They combined the non-instantaneous deterioration concept with sustainable investment and pricing strategies. Discount on selling price of products increases the selling; regarding this idea Rabbani et al.[234] established an inventory strategy for non-instantaneous deteriorating products in which the demand rate is dictated by inventory quality and price fluctuations over time. Mashud et al.[235] created joint pricing deteriorating inventory model considering product life cycle and advance payment with a discount facility.

Inventory models on markdown policy: Businesses have challenges throughout the cycle because they must strike a balance between the need to maximize profit and the need to get

clear of end-of-life inventory. The next step is the establishment of a markdown policy to decrease waste as fresh products get closer to the end of their selling time. Urban and Baker[236] examined how much of a markdown on the selling price might be applied during the deterioration period in order to enhance profit per unit of time as well as how a pre-deterioration markdown might affect the unit profit. Wee and Widyadana[237] created a deteriorating inventory model under a markdown policy to increase the profit, with nonlinear price-dependent demand. Srivastava and Gupta[238] formulated an EPQ deteriorating model with price and time-dependent demand under a markdown policy to reduce inventory level at end of cycle time for clearing the stocks and increase profit. In the context of a markdown policy, Kamaruzaman and Omar[81] created an inventory model for a fresh product whose demand is influenced by its price, level of inventory, freshness, and expiration date. Nurzahara and Kamaruzaman[239] developed the inventory models with markdown policy under different demand structures. Singh and Rani[240], Shee and Chakrabarti[241], Roy[242]; etc. authors were constructed economic production quantity model for deteriorating products under markdown policy.

2.6 Research gap

As we discussed the different literatures related to our proposed work, none has considered the following points: the scope of our proposed research work is to consider the following new ideas, which has not been taken up by any researcher in the study of inventory management.

➤ **Research gap for the inventory models of the "new and used buyback products"**

- (i) The demand of new products is a nonlinear form of selling price and exponential decreasing form of time in the modelling of new and used products inventory. The demand of used buyback product is a linear form of selling price and time.
- (ii) The inventory model of new and used buyback products with demand as per (i) with deterioration; and rate of deterioration of used buyback product is more than new products. The effect of deterioration of new product and used products on retailer's decisions.
- (iii) The inventory model of new and used buyback products with partially backlogging shortages. The demand during positive cycle time is same as (i) and demand during

shortages period for new products is non linear function of selling price and for used buyback products is linear function of selling price.

- (iv) The inventory model of new and used buyback products with deterioration and partially backlogging shortages. Deterioration effects on total profit in case of shortages.

➤ **Research gap for the inventory models with carbon emissions and green investments**

- (v) An EOQ model with price and green investment (as a carbon reduction function) dependent under time dependent deterioration rate; Carbon tax, cap-trade carbon policy with trade credit financing.

- (vi) A sustainable economic production model with price and green investment(as a carbon reduction function) dependent under expiration date; adopted a tax-cap carbon policy.

- (vii) The VMI model and traditional model for non-instantaneous deterioration products with green investment (as a carbon reduction function) and promotion level-dependent demand included shortages.

➤ **Research gap for the inventory models that incorporated freshness and greening efforts for perishable products**

- (viii) The optimal policies for non-instantaneous deteriorating perishable products with the concept of greening efforts, freshness and price-related demand, and price discount policy.

- (ix) An EPQ model for delay deteriorating perishable products with greening efforts, freshness and price-related demand and markdown strategy.

2.7 Objective of study

We outline the following as the main objectives of the study:

- To derive optimal order quantities of new products and used buyback quantity, i.e. to obtain refill policy of new product and take back policy of used product.
- To identify the effect of price discount facility on buyback used product on retailer's profit.
- To identify the effect of the deterioration of new products and used buyback products on total profit.

- To investigate the positive cycle time and shortages period and to derive the impact of a backlogging rate on the total profit of the retailer in new and buyback used product's inventory.
- To optimize the selling price and replenishment cycle time such that the retailer's total profit is maximized by trading of new as well as used products.
- To develop the sustainable EOQ model with green investment and price-dependent demand with carbon tax, cap and trade policy under trade credit financing. To investigate which carbon policy is better for retailers. To identify the role of green technology investment that is helpful to minimize carbon emissions. Investigate the optimum value of selling price, replenishment cycle time and green investment cost such that the retailer's total profit is maximized.
- To create a SEPQ model for products that incorporates several practical features such as green investment and price-sensitive demand, a time-varying deterioration rate, an expiration date, as well as a carbon cap and tax policy. To derive the optimal value of selling price and green investment cost such that the manufacturer's total profit is maximized.
- To developed the traditional inventory model and VMI model with green investment and its promotion level dependent demand with non instantaneous deterioration and partially backlogged shortages. Obtain results of the traditional inventory model and VMI model, to identify VMI model is better than traditional model. To optimize value of cycle time and green investment cost such that the supply chain total cost is minimized.
- To design and analyze the inventory model considering the selling price, freshness (age of product), and greening efforts related to demand with physical and quality base deterioration, including a selling price discount.
- To optimize the producer's total profit maximize the optimum value of selling price, cycle time, and markdown percentage considering demand dependent on selling price, freshness (age of product), and greening efforts. To obtain optimum markdown offering time, the quantity of non-deteriorating products and markdown offering quantity.

CHAPTER-3

Retailer's Optimal Inventory Decisions for New Products and a Buyback Decision for Used Products

3.0 Introduction

In today's market scenario, people are not only interested in buying a newly produced or recently launched product, but due to environmental awareness and because it has the same features and is cheaper than a new product; they are also willing to buy a used product from a retailer by Chen et al.[138]. Some instances of long-standing practices include scrap metal traders, recyclable paper, and buyback schemes for beverage bottles, mobile phones, marker pens, batteries, and electronics products. In these situations, recovering the used products is more advantageous financially than throwing them away. Reusing products has sparked more curiosity as awareness of environmental issues increases Koh et al. [146]. Utilizing used products after they have completed their useful life spans can help conserve limited natural resources and generate revenue for businesses Denial et al. [147] . Currently, many firms and online shopping sites like Amazon, Flipkart, and others provide refurbished products with the same features as newly launched products with a price discount facility. We explore reusable products with a straightforward buyback mechanism. In order to satisfy demand, the retailer has two options: either buy back used products and return them as new products, or order new products from outside suppliers[146]. Our study optimized the quantity of new products and the buyback quantity of used products simultaneously to satisfy the demand. Shortages are not allowed. The price of products and time are major factors in demand; a higher selling price may decrease demand with time. Product deterioration is a natural factor that

directly affects inventory decisions, and it should never be overlooked during the modelling of an inventory system, Whitin [4], Ghare and Schrader [5].

In this chapter, we discuss two inventory models from the retailer's point of view that focus on inventory policies with cases without deterioration and with deterioration. Our study focused on the, (i) a retailer sells new products as well as collects and sells used products to customers. The retailer satisfies market demand by selling new products as well as used products back from customers, (ii) the rate of demand is assumed to be a nonlinear decreasing function of price, an exponentially decreasing function of time for new products, and a linearly decreasing function of price and time for buy back used products, (iii) developed two inventory models and analyse the effect of deterioration on total profit. The objective is to maximize total profit for the retailer with respect to optimal price, cycle time, and ordering quantity for new products and optimal buyback quantity for used products simultaneously. The optimality of an objective function is verified theoretically, by the hessian matrix method, and graphically. A numerical example has been looked at to demonstrate the feasibility of the models. Sensitivity analysis has highlighted the management implications for the most realistic opportunity with respect to parameters. There are also a few closing remarks and future scopes of study provided. Two inventory models are established in this chapter as below:

Model-3.1 Optimal inventory decision for non-deteriorating products

Model-3.2 Optimal inventory decision for deteriorating product

3.1 Optimal inventory decision for non-deteriorating products

In this section, we presume that the retailer sells products that are non-deteriorating during the cycle time. Demand for non-deteriorating products is time- and price-sensitive for new products as well as buy-backs of used products. The retailer offers a price discount on the used products that are buy-back from customers. Our aim is to maximize the total profit of the retailer with respect to the replenishment cycle time and selling price and optimize the quantity of new products and the buyback quantity of used products without considering deterioration.

The proposed inventory model 3.1 is formulated using following notations and assumptions:

3.1.1 Notations and Assumptions

3.1.1.1 Notations

Parameters

A	Retailer ordering cost (in ₹/order).
C	Purchase cost (Constant) (in ₹/unit).
h	Inventory holding cost (in ₹/unit) for new product.
h_u	Inventory holding cost (in ₹/unit) for used buy back product.
τ	The point of time when collection and sell of used buy back products starts (years), $0 \leq \tau \leq T$.
p_0	Discount rate on selling price for used product.
d_r	Rate of depreciation on purchase cost for used buy back product.

Decision variables

p	Selling Price (in ₹/unit) (a decision variable).
T	The length of ordering cycle (a decision variable) (years).

Objective function

$TP(p, T)$ Total profit function of the retailer which is the sum of profit generated from new products and buy-back used products (in ₹).

Expressions and functions

$R_n(p, t)$	Demand rate for new product at $0 \leq t \leq T$ (units).
$R_u(p, t)$	Demand rate for used product at $\tau \leq t \leq T$ (units).
$I(t)$	Inventory level at time $0 \leq t \leq T$ for new product (units).
$I_u(t)$	Inventory level at time $\tau \leq t \leq T$ for used buy back product (units).
Q	The replenishment quantity for new product.
Q_u	Buy-back quantity of used products.
$TP_n(p, T)$	Profit generated from new products (in ₹).
$TP_u(p, T)$	Profit generated from buy-back used products (in ₹).

3.1.1.2 Assumptions

1. The inventory system deals with single product.
2. The replenishment is instantaneous and planning horizon is infinite.
3. The holding cost is considered to be constant for new product as well as used buyback product with $h > h_u$.
4. The rate of demand for new product is taken as a $R_n(p, t) = ap^{-b}e^{-\varepsilon t}$, $0 \leq t \leq T$, where $a > 0$ denotes the scale demand, $0 < b < 1$ denotes the price elasticity and $0 < \varepsilon < 1$.
5. The buyback rate of used product is taken as, $R_u(p, t) = \alpha(1 - \beta t) - p(1 - p_0)$, $\tau \leq t \leq T$, where $\alpha > 0$ and $0 < \beta < 1$.
6. The Lead time is negligible or zero and shortages are not allowed.
7. A retailer sells the new product to customers as well as collects and sells the used products again. The collection of used products and selling the both type of products are simultaneously during the cycle time.
8. Retailer offers a price discount on used products.
9. Rework or repairing of used products is not considered.
10. A retailer sells the new product during $0 \leq t \leq T$ and collects the used product at time τ and sell the used buyback product during $\tau \leq t \leq T$.

3.1.2 Mathematical Formulation

In this section, we outline the mathematical explanations and optimum results of the inventory model for both new and used products. Suppose that retailer have a maximum stock Q of new product at the starting the cycle time, the inventory level of the new products will be decreases due to the effect of demand, and hence the inventory level at time t over the period $[0, T]$ can be represented by the differential equation,

$$\frac{dI(t)}{dt} = -R_n(p, t), 0 \leq t \leq T \quad (3.1.1)$$

At time $t = T$, the inventory level approaches to zero. The solution of the differential equation (3.1.1) is given by,

$$I(t) = \frac{ap^{-b}}{\varepsilon} (e^{-\varepsilon t} - e^{-\varepsilon T}), 0 \leq t \leq T \quad (3.1.2)$$

But $I(0) = Q$ gives that,

$$Q = \frac{ap^{-b}}{\varepsilon} (1 - e^{-\varepsilon T}) \quad (3.1.3)$$

Now, for the used product on the span $[\tau, T]$, the used product return rate has an impact on the inventory level. So the differential equation for inventory level $I_u(t)$ at any time t ,

$$\frac{dI_u(t)}{dt} = -R_u(p, t), \tau \leq t \leq T \quad (3.1.4)$$

But used product inventory level also reached zero at time $t = T$, solution of (3.1.4) at $I_u(T) = 0$ is,

$$I_u(t) = \alpha \left((T - t) - \frac{\beta}{2} (T^2 - t^2) \right) - p(1 - p_0)(T - t) \quad (3.1.5)$$

Thus, the buy-back quantity of used products is,

$$Q_u = p(1 - p_0)(T - \tau) - \alpha \left((T - \tau) - \frac{\beta}{2} (T^2 - \tau^2) \right) \quad (3.1.6)$$

In order to determine the new product's total profit, we now compute the sales revenue and relevant costs.

$$\text{Sales revenue from new product: } SR_n = \frac{p}{T} \left(\int_0^T ap^{-b} e^{-\varepsilon t} dt \right) \quad (3.1.7)$$

$$\text{Purchase cost of new product: } PC_n = \frac{CQ}{T} \quad (3.1.8)$$

$$\text{Holding cost for new product: } HC_n = \frac{1}{T} \int_0^T [h \cdot I(t)] dt \quad (3.1.9)$$

$$\text{Ordering cost: } OC_n = \frac{A}{T} \quad (3.1.10)$$

So, the total profit generated from new products during the cycle is from (3.1.7) to (3.1.10)

$$TP_n(p, T) = SR_n - OC_n - HC_n - PC_n \quad (3.1.11)$$

On the other side, to determine the total profit from the used products, we now compute the sales revenue and relevant costs for used product.

$$\text{Revenue from selling used product: } SR_u = \frac{p(1-p_0)}{T} \left(\int_{\tau}^T (\alpha(1-\beta t) - p(1-p_0)) dt \right) \quad (3.1.12)$$

$$\text{Purchase cost for used product: } PC_u = \frac{C(1-d_r)Q_u}{T-\tau} \quad (3.1.13)$$

$$\text{Cost of holding of used product: } HC_u = \frac{1}{T} \int_{\tau}^T [h_u \cdot I_u(t)] dt \quad (3.1.14)$$

Thereupon, the total profit for used product during the cycle is,

$$TP_u(p, T) = SR_u - HC_u - PC_u \quad (3.1.15)$$

Hence, the total profit of the retailer generated from the new products and buyback used products is from (3.1.11) and (3.1.15),

$$\begin{aligned} TP(p, T) &= TP_n(p, T) + TP_u(p, T) \\ &= \frac{ap^{(1-b)}(1-e^{-\varepsilon T})}{T\varepsilon} - \frac{A}{T} - \frac{ha(e^{\varepsilon T} - \varepsilon T - 1)e^{-\varepsilon T}p^{-b}}{\varepsilon^2 T} - \frac{Cap^{-b}(1-e^{-\varepsilon T})}{T} \\ &\quad + \frac{1}{T} \left(p(1-p_0)(\alpha - p(1-p_0))(T-\tau) - \frac{1}{2}(p(1-p_0)\alpha\beta(T^2 - t^2)) \right) \\ &\quad - \frac{h_u}{T} \left(\frac{1}{6}\alpha\beta(T^3 - \tau^3) + \frac{1}{2}(-\alpha + p(1-p_0))(T^2 - \tau^2) \right. \\ &\quad \left. + (T-\tau)\alpha(T - \frac{1}{2}\beta T^2) - p(1-p_0)T \right) \\ &\quad - \frac{C(1-d_r)}{T-\tau} \left(p(1-p_0)(T-\tau) - \alpha(T-\tau - \frac{1}{2}\beta(T^2 - t^2)) \right) \end{aligned} \quad (3.1.16)$$

The total profit is a function of selling price p and the replenishment cycle time T . The objective is to find the optimal selling price and the replenishment cycle time such that the retailer's total profit is maximized.

3.1.2.1 Solution technique to determine the optimal solution

According to R. Sundararajan et al.[70], the solution approach for determining the optimum value of the decision variables for this proposed model is used. To obtain the optimal selling price that corresponds to maximising the total profit, for given T , we first check necessary and sufficient conditions. The necessary condition for finding the optimal selling price p^* for fix value of T is given as follows:

$$\begin{aligned} \frac{\partial TP(p,T)}{\partial p} = & -\frac{ap^{1-b}(1-b)}{Tp\varepsilon}[e^{-T\varepsilon}-1] - \frac{Cap^{-b}b}{Tp\varepsilon}[e^{-T\varepsilon}-1] + \frac{haby^{-b}e^{-T\varepsilon}}{Tp\varepsilon^2}[e^{T\varepsilon}\varepsilon - \varepsilon T - 1] \\ & + \frac{(1-p_0)}{T} \left[-\frac{1}{2}\alpha\beta(T^2 - \tau^2) + (\alpha - p(1-p_0))(T - \tau) - p(1-p_0)(T - \tau) \right] \\ & - C(1-d_r)(1-p_0) - \frac{h_u(1-p_0)}{2T}[(T^2 - \tau^2) - 2(T - \tau)] = 0 \end{aligned} \quad (3.1.17)$$

Theorem 3.1.1: *For a given value of T , we have*

(a) Equation (3.1.17) has one and only one solution.

(b) The sufficient conditions for maxima satisfied by the value of p obtained in (a).

Proof: Taking second-order derivative of (3.1.16) with respect to p and simplifying term is given as,

$$\begin{aligned} \frac{\partial^2 TP(p,T)}{\partial p^2} = & \frac{abp^{1-b}(e^{-\varepsilon T}-1)(1-b)}{Tp^2\varepsilon} - \frac{haby^{-b}(1-e^{-\varepsilon T}-\varepsilon Te^{-\varepsilon T})(b+1)}{Tp^2\varepsilon^2} \\ & + \frac{Cabp^{-b}(e^{-\varepsilon T}-1)(b+1)}{Tp^2\varepsilon} - \frac{2(1-p_0)^2(T-\tau)}{T} \end{aligned}$$

But in above expression, we have $e^{-\varepsilon T}-1 < 0$, $1-e^{-\varepsilon T}-\varepsilon Te^{-\varepsilon T} > 0$ and $T > \tau$, that means

$$\frac{\partial^2 TP(p,T)}{\partial p^2} < 0 \quad (3.1.18)$$

Hence, p^* is one and only one solution of (3.1.17) for the given fix positive value of T and objective function satisfied the second-order condition for the maximum at p^* for fixed positive T .

Now, to obtain the optimal cycle time that correspond to maximizing the total profit, for given fix selling price, we first check necessary and sufficient conditions. The necessary condition for finding the optimal cycle time T^* for fix value of p is given as follows:

$$\begin{aligned}
\frac{\partial TP(p,T)}{\partial T} &= \frac{ab^{1-b}}{T^2 \varepsilon} [e^{-\varepsilon T} - 1] + \frac{ab^{1-b} e^{-\varepsilon T}}{T} + hap^{-b} \left[\frac{(-\varepsilon T + e^{\varepsilon T} - 1)}{T^2 \varepsilon^2} \right. \\
&\quad \left. - \frac{(-\varepsilon + \varepsilon e^{\varepsilon T})}{T \varepsilon^2} + \frac{(-\varepsilon T + e^{\varepsilon T} - 1)}{T \varepsilon} \right] - Cap^{-b} \left[\frac{e^{-\varepsilon T}}{T} + \frac{(e^{-\varepsilon T} - 1)}{\varepsilon T^2} \right] \\
&\quad + \frac{A}{T^2} + \Upsilon_1 - \Upsilon_2 + h_u [\Upsilon_3 - \Upsilon_4] \\
&\quad + \frac{C(1-d_r)}{(T-\tau)^2} \left[\begin{aligned} &\left(p(1-p_0)(T-\tau) - \alpha(T-\tau - \frac{1}{2}\beta(T^2 - \tau^2)) \right) \\ &- (p(1-p_0)(T-\tau) - \alpha(-T\beta + 1)(T-\tau)) \end{aligned} \right] = 0
\end{aligned} \tag{3.1.19}$$

In the mathematical expression (3.1.19) $\Upsilon_1, \Upsilon_2, \Upsilon_3, \Upsilon_4$ are as,

$$\begin{aligned}
\Upsilon_1 &= \left(\frac{-p(1-p_0)\alpha\beta T + p(1-p_0)(\alpha - p(1-p_0))}{T} \right), \\
\Upsilon_2 &= \left(\frac{-\frac{1}{2}p(1-p_0)\alpha\beta(T^2 - \tau^2) + p(1-p_0)(\alpha - p(1-p_0))(T-\tau)}{T^2} \right), \\
\Upsilon_3 &= \left(\frac{\frac{1}{6}\alpha\beta(T^3 - \tau^3) + \frac{1}{2}(-\alpha + p(1-p_0))(T^2 - \tau^2) + \alpha((T - \frac{1}{2}\beta T^2) - p(1-p_0))(T-\tau)}{T^2} \right), \\
\text{and } \Upsilon_4 &= \frac{1}{T^2} \left(\begin{aligned} &\frac{1}{2}\alpha\beta T^2 + (-\alpha + p(1-p_0))T + \alpha((-T\beta + 1) - p(1-p_0))(T-\tau) \\ &+ \alpha((T - \frac{1}{2}\beta T^2) - p(1-p_0))T \end{aligned} \right)
\end{aligned}$$

Theorem 3.1.2: For a given value of p , we have

- (a) Equation (3.1.19) has one and only one solution.
- (b) The sufficient conditions for maxima satisfied by the value of T obtained in (a).

Proof: Taking second-order partial derivative of (3.1.16) with respect to T and simplifying terms is given as follows:

$$\begin{aligned}
\frac{\partial^2 TP(p, T)}{\partial T^2} = & -\frac{2ap^{1-b}}{T^3 \varepsilon} \left[(1 + T\varepsilon + \frac{T^2 \varepsilon^2}{2}) e^{-\varepsilon T} - 1 \right] - \frac{2A}{T^3} \\
& - \frac{2ap^{-b}}{T} \left[(1 - e^{-\varepsilon T}) \left(\frac{C}{\varepsilon T^2} - \frac{h}{T\varepsilon} - h \right) - C \left(\frac{1}{T} + \frac{\varepsilon}{2} \right) e^{-\varepsilon T} \right] \\
& - hap^{-b} (1 - (\varepsilon T - 1)e^{-\varepsilon T}) \left[\frac{1}{T} + \frac{2}{T^3 \varepsilon^2} + \frac{2}{T^2 \varepsilon} \right] - \frac{p(1-p_0)}{T^3} \left[2(\alpha - p(1-p_0))\tau - \alpha\beta(T^2 + \tau^2) \right] \\
& - \frac{C(1-d_r)}{(T-\tau)^2} [0] - \frac{2h_u}{T^2} \left[\left(\frac{\alpha\beta Tp(1-p_0)}{2h_u} \right) - (\alpha\beta T(T-\tau+1) + (T+\tau)(p(1-p_0) - \alpha)) \right] \\
& - \frac{2h_u}{T^3} \left(\frac{\alpha\beta}{6} (T^3 - \tau^3) + \frac{1}{2} (p(1-p_0) - \alpha)(T^2 - \tau^2) + \left(\alpha(T - \frac{1}{2}\beta T^2) - p(1-p_0)T \right) (T - \tau) \right)
\end{aligned}$$

For notation convenience, let's take,

$$\begin{aligned}
\Psi_1 &= (1 + T\varepsilon + \frac{T^2 \varepsilon^2}{2}) e^{-\varepsilon T} - 1, \Psi_2 = (1 - e^{-\varepsilon T}) \left(\frac{C}{\varepsilon T^2} - \frac{h}{T\varepsilon} - h \right) - C \left(\frac{1}{T} + \frac{\varepsilon}{2} \right) e^{-\varepsilon T}, \\
\Psi_3 &= 1 - (\varepsilon T - 1)e^{-\varepsilon T}, \Psi_4 = 2(\alpha - p(1-p_0))\tau - \alpha\beta(T^2 + \tau^2), \\
\Psi_5 &= \left(\frac{\alpha\beta Tp(1-p_0)}{2h_u} \right) - (\alpha\beta T(T-\tau+1) + (T+\tau)(p(1-p_0) - \alpha)), \\
\Psi_6 &= \frac{\alpha\beta}{6} (T^3 - \tau^3) + \frac{1}{2} (p(1-p_0) - \alpha)(T^2 - \tau^2) + \left(\alpha(T - \frac{1}{2}\beta T^2) - p(1-p_0)T \right) (T - \tau)
\end{aligned}$$

For the conditions $T \geq 0$, $T > \tau$, $p > C$ and $a > 0$, $0 < b < 1$, $0 < \varepsilon < 1$, $\alpha > 0$, $0 < \beta < 1$

We have,

$$\begin{aligned}
\Psi_1 &> 0 \text{ for } (1 + T\varepsilon + \frac{T^2 \varepsilon^2}{2}) e^{-\varepsilon T} > 1, \Psi_2 > 0 \text{ for } (1 - e^{-\varepsilon T}) \left(\frac{C}{\varepsilon T^2} - \frac{h}{T\varepsilon} - h \right) > C \left(\frac{1}{T} + \frac{\varepsilon}{2} \right) e^{-\varepsilon T}, \\
\Psi_3 &> 0 \text{ for } 1 - (\varepsilon T - 1)e^{-\varepsilon T} > 0, \Psi_4 > 0 \text{ for } 2(\alpha - p(1-p_0))\tau > \alpha\beta(T^2 + \tau^2), \\
\Psi_5 &> 0 \text{ for } \left(\frac{\alpha\beta Tp(1-p_0)}{2h_u} \right) > (\alpha\beta T(T-\tau+1) + (T+\tau)(p(1-p_0) - \alpha)) \text{ and} \\
\frac{\alpha\beta}{6} (T^3 - \tau^3) &+ \frac{1}{2} (p(1-p_0) - \alpha)(T^2 - \tau^2) + \left(\alpha(T - \frac{1}{2}\beta T^2) - p(1-p_0)T \right) (T - \tau) > 0 \quad \text{gives} \\
\Psi_6 &> 0. \text{ It is clear that}
\end{aligned}$$

$$\frac{\partial^2 TP(p, T)}{\partial T^2} < 0 \tag{3.1.20}$$

Hence, T^* is a unique solution of (3.1.19) for the fixed p^* and objective function satisfied the second-order condition for the maximum at T^* for fixed positive p^* .

3.1.3 Numerical experiment

The suggested model is demonstrated using the example given below:

Example 3.1.1: The following numerical values of the parameters in proper unit were considered as input for numerical, graphical and sensitivity analysis of the model.

The scale demand of new product $a = 255$ units, price elasticity of new product $b = 0.4$, $\varepsilon = 0.9$, $\alpha = 100$, $\beta = 0.3$, purchasing cost $C = ₹55$ per unit, ordering cost $A = ₹100$ per order, holding cost of new product $h = ₹0.5/\text{unit/year}$, holding cost of used buyback product $h_u = ₹0.2/\text{unit/year}$, rate of depreciation of buyback product $d_r = 0.15$, $\tau = \frac{30}{365}$ year, price discount on selling price of used buyback product $p_0 = 0.5$.

Using mathematical software like, or maple 18 or MATLAB the optimal results of proposed model given in Table 3.1, and validation of sufficient conditions by hessian matrix method are given below:

Table 3.1 Optimal results of proposed model 3.1

p^* (in ₹)	T^* (year)	Q^* (units)	Q_u^* (units)	Total Profit (in ₹)
103.8575	0.3698	12.52	11.88	4978.10

The concavity of the profit function is developed by the well-known hessian matrix.

Let's consider the following hessian matrix,

$$H(p, T) = \begin{pmatrix} \frac{\partial^2 TP(p, T)}{\partial p^2} & \frac{\partial^2 TP(p, T)}{\partial p \partial T} \\ \frac{\partial^2 TP(p, T)}{\partial T \partial p} & \frac{\partial^2 TP(p, T)}{\partial T^2} \end{pmatrix} \quad (3.1.21)$$

The hessian matrix at the optimal value of decision variable is,

$$H(p^*, T^*) = \begin{pmatrix} -0.5639599422 & -20.64316302 \\ -20.64316302 & -11439.53819 \end{pmatrix} \quad (3.1.22)$$

If the Eigen values of the Hessian matrix at the solution (p^*, T^*) are all negative then the profit function $TP(p^*, T^*)$ is maximum at the solution (Cárdenas-Barron and Sana[203]). Here, Eigen values of the hessian matrix (3.1.22) are $\lambda_1 = -11439.5$, $\lambda_2 = -0.5267$. Therefore, the profit function $TP(p^*, T^*)$ is maximized at (p^*, T^*) .

From above Hessian Matrix (3.1.21), define that $\Delta_{11} = \frac{\partial^2 TP(p, T)}{\partial p^2}$, $\Delta_{22} = \frac{\partial^2 TP(p, T)}{\partial T^2}$ & $\Delta_{12} = \frac{\partial^2 TP(p, T)}{\partial p \partial T}$ for optimal value of p^* and T^* , it is clear, from (3.1.22) $\Delta_{11} = -0.5639599422 < 0$, $\Delta_{22} = -11439.53819 < 0$ and $\Delta_{11}\Delta_{22} - (\Delta_{12})^2 = 6025.3 > 0$ then the optimal value of p^* and T^* satisfies (3.1.17) and (3.1.19), value of p^* and T^* is unique and maximize $TP(p, T)$.

3.1.3.1 Graphical authentication of the concavity of objective functions

The concavity of profit function is presented in Figure 3.1, Figure 3.2 and Figure 3.3 as below:

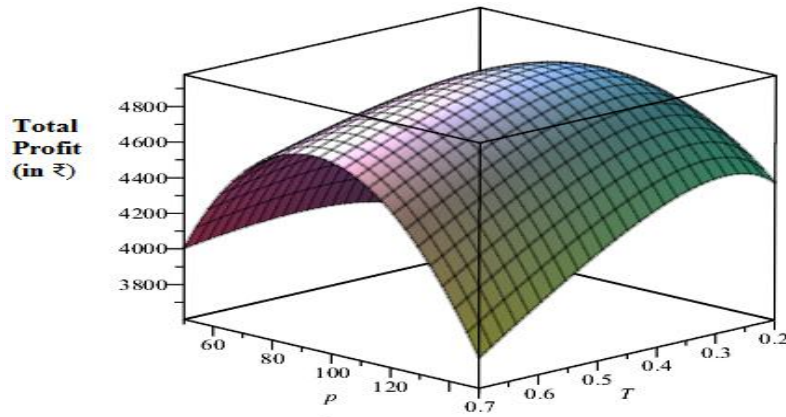


Figure 3.1 Concavity of total profit function with respect to p and T for model 3.1

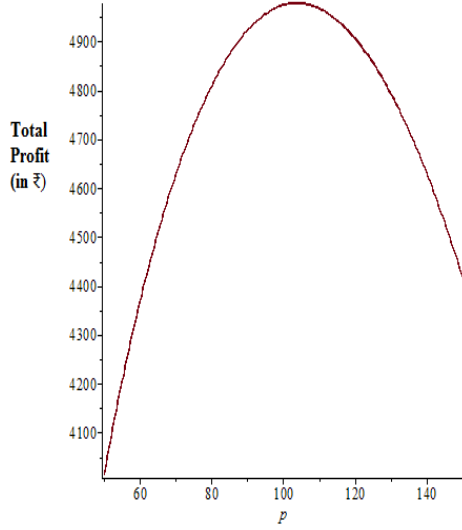


Figure 3.2 Total profit vs selling price for model 3.1

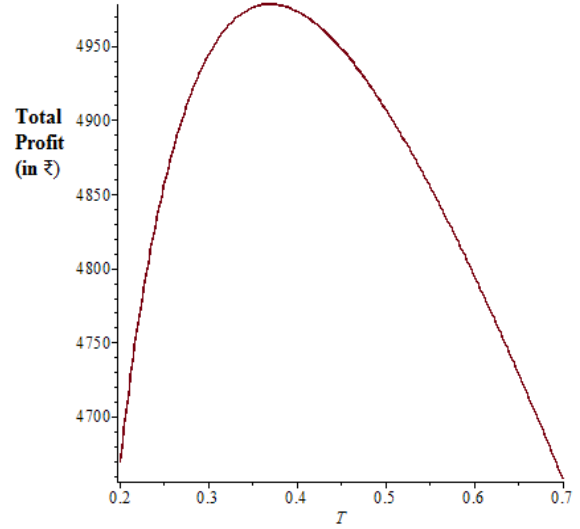


Figure 3.3 Total profit vs cycle time for model 3.1

3.1.4 Sensitivity Analysis and observations

In this section, using the mathematical software Maple 18 or Matlab, we evaluated the sensitivity effect on the optimal results. The sensitivity analysis is carried out by changing the values of the parameters specified in Example 3.1.1 in proportional steps of -20%, -10%, +10%, and +20%, taking each parameter one at a time while the remaining parameter values remain untouched.

Table 3.2 Sensitivity with respect to key parameters

Inventory Parameter	Change %	Values	T^* (Year)	p^* (in ₹)	Q^* Units	Q_u^* Units	Total Profit (in ₹)	Eigen Values of (3.1.21) (λ_1, λ_2)
a	-20	204	0.4021	92.83	11.24	14.82	4680.76	(-14678.4, -0.5538)
	-10	229.5	0.3859	98.37	11.94	13.30	4821.11	(-10111.7, -0.5350)
	0	255	0.3698	103.86	12.52	11.88	4978.10	(-11439.5, -0.5267)
	10	280.5	0.3538	109.31	13.00	10.54	5151.26	(-12907.3, -0.5285)
	20	306	0.3381	114.77	13.38	9.29	5340.26	(-14668.4, -0.5057)
b	-20	0.32	0.2858	135.26	13.36	5.47	5948.68	(-22437.21, -0.4908)
	-10	0.36	0.3335	116.98	13.23	8.86	5358.99	(-15207.62, -0.5330)
	0	0.4	0.3698	103.86	12.52	11.88	4978.10	(-11439.5, -0.5267)
	10	0.44	0.3982	93.83	11.57	14.50	4726.21	(-9212.12, -0.5882)
	20	0.48	0.4204	85.91	10.53	16.74	4557.53	(-7795.78, -0.6077)
α	-20	80	0.3665	91.63	13.07	8.19	3399.35	(-8805.77, -0.6122)

Inventory Parameter	Change %	Values	T^* (Year)	p^* (in ₹)	Q^* Units	Q_u^* Units	Total Profit (in ₹)	Eigen Values of (3.1.21) (λ_1, λ_2)
	-10	90	0.3677	97.63	12.77	10.02	4177.53	(-10115.86,-0.5860)
	0	100	0.3698	103.86	12.52	11.88	4978.10	(-11439.5,-0.5267)
	10	110	0.3726	110.30	12.30	13.75	5802.03	(-12780.10,-0.5152)
	20	120	0.3758	116.95	12.10	15.64	6650.24	(-14140.53,-0.5093)
β	-20	0.24	0.3997	103.79	13.37	13.44	5100.56	(-9013.07,-0.5702)
	-10	0.27	0.3839	103.83	12.92	12.61	5038.20	(-10205.44,-0.5670)
	0	0.3	0.3698	103.86	12.52	11.88	4978.10	(-11439.5,-0.5267)
	10	0.33	0.3572	103.87	12.16	11.22	4920.01	(-12714.14,-0.5109)
	20	0.36	0.3458	103.88	11.83	10.64	4863.74	(-14028.15,-0.5001)
C	-20	44	0.3567	110.44	11.85	10.49	4975.18	(-12552.03,-0.5242)
	-10	49.5	0.3633	107.05	12.19	11.19	4976.90	(-11977.32,-0.5233)
	0	55	0.3698	103.86	12.52	11.88	4978.10	(-11439.5,-0.5267)
	10	60.5	0.3762	100.84	12.85	12.55	4987.38	(-10934.44,-0.5275)
	20	66	0.3824	97.99	13.18	13.22	5001.41	(-10458.61,-0.5295)
A	-20	80	0.3559	104.37	12.10	11.29	5033.22	(-11983.69,-0.5589)
	-10	90	0.3629	104.11	12.31	11.58	5005.39	(-11704.71,-0.5315)
	0	100	0.3698	103.86	12.52	11.88	4978.10	(-11439.5,-0.5267)
	10	110	0.3766	103.61	12.73	12.16	4951.30	(-11188.80,-0.5100)
	20	120	0.3833	103.37	12.93	12.45	4924.98	(-10950.72,-0.5010)
h	-20	0.4	0.3699	103.85	12.53	11.88	4978.69	(-11429.29,-0.5266)
	-10	0.45	0.3699	103.85	12.52	11.88	4978.39	(-11434.41,-0.5267)
	0	0.5	0.3698	103.86	12.52	11.88	4978.10	(-11439.5,-0.5267)
	10	0.55	0.3697	103.86	12.52	11.87	4977.80	(-11444.7,-0.5267)
	20	0.6	0.3697	103.87	12.52	11.87	4977.50	(-11450.10,-0.5267)
h_u	-20	0.16	0.3699	103.85	12.52	11.88	4978.27	(-11433.5,-0.5267)
	-10	0.18	0.3698	103.85	12.52	11.88	4978.19	(-11436.5,-0.5267)
	0	0.2	0.3698	103.86	12.52	11.88	4978.10	(-11439.5,-0.5267)
	10	0.22	0.3698	103.86	12.52	11.88	4978.01	(-11441.3,-0.5267)
	20	0.24	0.3697	103.86	12.52	11.88	4977.92	(-11444.3,-0.5267)
d_r	-20	0.12	0.3703	102.38	12.61	12.11	5046.83	(-11414.60,-0.5329)
	-10	0.135	0.3701	103.12	12.56	11.99	5012.31	(-11426.70,-0.5297)
	0	0.15	0.3698	103.86	12.52	11.88	4978.10	(-11439.50,-0.5267)
	10	0.165	0.3695	104.60	12.48	11.76	4944.18	(-11453.40,-0.5255)
	20	0.18	0.3693	105.35	12.44	11.64	4910.57	(-11469.35,-0.5243)
τ	-20	0.0658	0.3428	104.58	11.71	11.52	5099.33	(-12511.00,-0.5442)
	-10	0.0740	0.3565	104.22	12.12	11.71	5037.54	(-11945.78,-0.5352)
	0	0.0822	0.3698	103.86	12.52	11.88	4978.10	(-11439.50,-0.5267)
	10	0.0904	0.3827	103.50	12.91	12.03	4920.78	(-10939.30,-0.5132)
	20	0.0986	0.3952	103.16	13.27	12.16	4865.39	(-10309.27,-0.5002)
p_0	-20	0.4	0.3973	83.50	14.51	13.46	4478.27	(-9313.19,-0.7823)
	-10	0.45	0.3839	92.61	13.53	12.69	4711.78	(-10274.10,-0.6480)
	0	0.5	0.3698	103.86	12.52	11.88	4978.10	(-11439.50,-0.5267)
	10	0.55	0.3548	118.08	11.48	10.99	5286.80	(-12398.87,-0.4102)

Inventory Parameter	Change %	Values	T^* (Year)	p^* (in ₹)	Q^* Units	Q_u^* Units	Total Profit (in ₹)	Eigen Values of (3.1.21) (λ_1, λ_2)
	20	0.6	0.3385	136.59	10.41	10.01	5651.96	(-13540.70,-0.3202)
ε	-20	0.72	0.3779	105.14	13.11	11.99	5032.15	(-10810.00,-0.5341)
	-10	0.81	0.3737	104.50	12.80	11.93	5004.57	(-11138.50,0.5304)
	0	0.9	0.3698	103.86	12.52	11.88	4978.10	(-11439.50,-0.5267)
	10	0.99	0.3663	103.23	12.26	11.84	4952.66	(-11710.70,-0.5127)
	20	1.08	0.3632	102.61	12.02	11.81	4928.19	(-1201.15,-0.5012)

The information provided in the numerical example is taken into account in order to observe how the inventory parameters affect an optimal solution.

- Here observed that Eigen values of hessian matrix (3.1.21) at corresponding value of p^* and T^* all are negative, that implies profit is maximize at (p^*, T^*) .
- System parameters a, α, d_r and p_0 are increases then optimal selling price increases but if parameters b, C, ε increases then selling price will be decrease. Yet, selling price almost unchanged for changes in holding cost parameters, higher value of τ gives lower selling price, but increases β then selling price decreases. Although, selling price is highly positive sensitive to a, α, d_r and p_0 and strongly negative sensitive to b, C, ε .
- When the value of the parameters a, α, C, τ and p_0 are increases, the optimal total profit will be increase. However, for increasing in parameters b, β, A, d_r and ε then the total profit will be decrease. Others parameters effects on total profit is minor.
- Replenishment cycle time is observed to be positively correlated with system parameters b, α, C, A, τ and negative related to $a, \beta, d_r, \varepsilon$. Nevertheless, changes in holding cost parameters don't have much of an impact on the length of the replenishment cycle.
- Optimal order quantity of new products will be increase if the parameters a, α, C, τ are increases but optimal order quantity of new product will be decreases if the parameters b, β and ε are increases.
- The buyback quantity of used products highly increases with increases in α and b .

3.2 Optimal inventory decision for deteriorating products

For the purpose of this section, it is assumed that the retailer sells the deteriorating products during the cycle time. New products as well as used products have a demand depending on time and selling price. The objective is to maximize the retailer's total profit while optimizing the quantity of new products and buy-back used products quantity, taking deterioration into account.

The proposed inventory model 3.2 is formulated using following notations and assumptions along with Section 3.1.1.

3.2.1 Notations and Assumptions

3.2.1.1 Notations

θ	Constant rate of deterioration for new product
θ_u	Constant rate of deterioration for used buyback product

3.2.1.2 Assumptions

The proposed model is constructed using the supplementary assumptions listed below.

1. The product's deterioration is considered, with a constant rate of deterioration for both new and used products. Rate of deterioration is θ for new product and θ_u is the rate of deterioration for used product with $\theta_u \geq \theta$.
2. Replacements or repairing for deteriorating products, during the cycle time will not be allowed.

3.2.2 Mathematical Formulation

As per the assumptions of this proposed model, the level of inventory decline with time due to combine effect of deterioration and demand. According to Ghare and Schrader[5] the level of inventory can be expressed by the differential equation in form of deterioration and demand function. So, the inventory level of the new products at the starting of

inventory cycle i.e. $t=0$ is maximum. Retailer's Stock level of decreases over the period $0 \leq t \leq T$ can be represented by the following differential equation,

$$\frac{dI(t)}{dt} + \theta I(t) = -R_n(p, t), 0 \leq t \leq T \quad (3.2.1)$$

At the end of inventory cycle (i.e. $t=T$), the inventory level goes to zero, the solution of (3.2.1) is at boundary condition $I(T) = 0$ is,

$$I(t) = \frac{ap^{-b}e^{-\theta t}}{\varepsilon - \theta} \left(e^{-t(\varepsilon - \theta)} - e^{-T(\varepsilon - \theta)} \right) \quad (3.2.2)$$

Maximum stock level of new products Q is available at time $t=0$, apply the condition $I(0) = Q$ in (3.2.2) the ordering quantity of new products is,

$$Q = \frac{ap^{-b}}{\varepsilon - \theta} \left(1 - e^{-T(\varepsilon - \theta)} \right) \quad (3.2.3)$$

The inventory level of buyback products is depends on the return rate used products. The retailer's inventory level of used products decreases due to demand and deterioration. Stock of used product $I_u(t)$ between $\tau \leq t \leq T$, can be expressed by differential equation given below at time t is,

$$\frac{dI_u(t)}{dt} + \theta_u I_u(t) = -R_u(p, t), \tau \leq t \leq T \quad (3.2.4)$$

The inventory level of used products approaches to zero at $t=T$, i.e. $I_u(T) = 0$.

The solution of (3.2.4) is,

$$I_u(t) = \frac{\alpha\beta t}{\theta_u} - \frac{\alpha\beta}{\theta_u^2} - \frac{\alpha}{\theta_u} + \frac{p}{\theta_u} (1 - p_0) - e^{\theta_u(T-t)} \left(\frac{\alpha\beta T}{\theta_u} - \frac{\alpha\beta}{\theta_u^2} - \frac{\alpha}{\theta_u} + \frac{p}{\theta_u} (1 - p_0) \right) \quad (3.2.5)$$

Thus, the quantity of buyback used product is given by,

$$Q_u = e^{\gamma(T-\tau)} \left(\frac{\alpha\beta T}{\theta_u} - \frac{\alpha\beta}{\theta_u^2} - \frac{\alpha}{\theta_u} + \frac{p}{\theta_u} (1 - p_0) \right) - \left(\frac{\alpha\beta \tau}{\theta_u} - \frac{\alpha\beta}{\theta_u^2} - \frac{\alpha}{\theta_u} + \frac{p}{\theta_u} (1 - p_0) \right) \quad (3.2.6)$$

To profit from new products, we must take into account all the factors listed below.

Sales revenue generated from selling the new product:

$$SR_n = \frac{p}{T} \left(\int_0^T ap^{-b} e^{-\varepsilon t} dt \right) \quad (3.2.7)$$

Purchase cost for new product:

$$PC_n = \frac{CQ}{T} \quad (3.2.8)$$

Carrying or holding for new product:

$$HC_n = \frac{1}{T} \int_0^T [h \cdot I(t)] dt \quad (3.2.9)$$

New product's ordering cost:

$$OC_n = \frac{A}{T} \quad (3.2.10)$$

Total profit for new product during the cycle is from (3.2.7) to (3.2.10),

$$TP_n(p, T) = SR_n - OC_n - HC_n - PC_n \quad (3.2.11)$$

To calculate total profit from buy back product, we calculate all the components listed as:

Sales revenue from buyback used products:

$$SR_u = \frac{p(1-p_0)}{T} \left(\int_{\tau}^T (\alpha(1-\beta t) - p(1-p_0)) dt \right) \quad (3.2.12)$$

Used product's purchase cost:

$$PC_u = \frac{C(1-d_r)Q_u}{T-\tau} \quad (3.2.13)$$

Used product's holding cost:

$$HC_u = \frac{1}{T} \int_{\tau}^T [h_u \cdot I_u(t)] dt \quad (3.2.14)$$

Total profit from used buyback product during the cycle is,

$$TP_u(p, T) = SR_u - HC_u - PC_u \quad (3.2.15)$$

Therefore, the total profit of retailer from (3.2.11) and (3.2.15) is,

$$\begin{aligned}
TP(p, T) = & \left(\frac{p}{T} \int_0^T [ap^{-b} e^{-\varepsilon t}] dt - \frac{CQ}{T} - \frac{1}{T} \int_0^T [h \cdot I(t)] dt - \frac{A}{T} \right) \\
& + \left(\frac{p(1-p_0)}{T} \int_{\tau}^T [(\alpha(1-\beta t) - p(1-p_0))] dt - \frac{C(1-d_r)Q_u}{T-\tau} - \frac{1}{T} \int_{\tau}^T [h_u \cdot I_u(t)] dt \right)
\end{aligned} \tag{3.2.16}$$

The total profit function is a function of selling price p and the replenishment cycle time T . The objective is to find the optimal selling price and the replenishment cycle time such that the retailer's total profit is maximized.

3.2.2.1 Solution technique to determine the optimal solution:

The solution procedure to evaluate the optimal value of decisions variables for this proposed model is adopted as per R.Sundararajan et. al [70].

To obtain the optimal selling price that corresponds to maximising the total profit, for given T , we first check necessary and sufficient conditions. The necessary condition for finding the optimal selling price p^* for fix value of T is given as follows:

$$\begin{aligned}
\frac{\partial TP(p, T)}{\partial p} = & \frac{ap^{-b}(e^{-T\varepsilon} - 1)}{T\varepsilon} [b-1] - \frac{Cap^{-b}b}{Tp(\varepsilon - \theta)} [1 - e^{-T(\varepsilon - \theta)}] \\
& + h \frac{abp^{-b}e^{-T\varepsilon}(e^{T\varepsilon}\theta - e^{T\theta}\varepsilon + \varepsilon - \theta)}{(\varepsilon - \theta)Tp\theta\varepsilon} \\
& + \frac{(1-p_0)}{T} \left(-\frac{1}{2}\alpha\beta(T^2 - \tau^2) + \alpha(T - \tau) - p(1-p_0)(T - \tau) \right) - \frac{p(1-p_0)^2(T - \tau)}{T} \\
& + \frac{C(1-d_r)}{T-\tau} \left(\frac{p_0}{\theta_u} - \frac{1}{\theta_u} + e^{\theta_u(T-\tau)} \left(-\frac{p_0}{\theta_u} + \frac{1}{\theta_u} \right) \right) \\
& - \frac{h_u(1-p_0)}{2T\theta_u^3} [2\theta_u^2(T - \tau) - 2e^{\theta_u(T-\tau)}\theta_u + 2\theta_u] = 0
\end{aligned} \tag{3.2.17}$$

Theorem 3.2.1: *Given fixed positive value of T then,*

- (a) *Equation (3.2.17) has one and only one solution.*
- (b) *The sufficient conditions for maxima satisfied by the value of p obtained in (a).*

Proof: To check the sufficient condition for optimal value of selling price, it is enough to show second order derivative of $TP(p, T)$ with respect to p , is less than zero.

$$\frac{\partial^2 TP(p, T)}{\partial p^2} = \frac{abp^{-b-1}(e^{-T\varepsilon} - 1)}{T\varepsilon} [1 - b] - \frac{Cap^{-b-2}}{T(\varepsilon - \theta)} (b^2 + b)[1 - e^{-T(\varepsilon - \theta)}] \\ - h \frac{ap^{-b-2}e^{-T\varepsilon}(e^{T\varepsilon}\theta - e^{T\theta}\varepsilon + \varepsilon - \theta)}{(\varepsilon - \theta)T\theta\varepsilon} (b^2 + b) - \frac{2(1 - p_0)^2(T - \tau)}{T}$$

In above expression, noticed that $e^{-T\varepsilon} - 1 < 0, 0 < b < 1, T \geq 0, T > \tau, p > C$ and $\varepsilon \neq \theta$.

$$\text{Clearly, } \frac{\partial^2 TP(p, T)}{\partial p^2} < 0. \quad (3.2.18)$$

Hence, (3.2.17) has one and only one solution and the sufficient condition for maxima satisfied by the optimal value of selling price.

Now, to obtain the optimal cycle time that correspond to maximising the total profit, for given fix selling price, we first check necessary and sufficient conditions.

The necessary condition for finding the optimal cycle time T^* for fix value of p is given as follows:

$$\begin{aligned}
\frac{\partial TP(p,T)}{\partial T} &= \frac{ap^{-b+1}}{T} \left[\left(\frac{e^{-T\varepsilon} - 1}{T\varepsilon} \right) + e^{-T\varepsilon} \right] + \frac{A}{T^2} \\
&+ \frac{Cap^{-b}}{T(\varepsilon - \theta)} \left[(-\varepsilon + \theta)e^{-T(\varepsilon - \theta)} + \frac{1}{T} - e^{-T(\varepsilon - \theta)} \right] \\
&+ \frac{hap^{-b}e^{-T\varepsilon}}{T\theta(\varepsilon - \theta)} \left[\frac{(e^{T\varepsilon} - 1)\theta - (e^{\theta T} - 1)\varepsilon}{T\varepsilon} - \theta(e^{T\varepsilon} - e^{\theta T}) \right. \\
&\quad \left. + \frac{(e^{T\varepsilon} - 1)\theta - (e^{\theta T} - 1)\varepsilon}{T\varepsilon} \right] \\
&+ \frac{p(1-p_0)}{T^2} \left[-\frac{1}{2}\alpha\beta(T^2 - \tau^2) + \alpha(T - \tau) \right. \\
&\quad \left. - p(1-p_0)(T - \tau) + T(-\alpha\beta T + \alpha - p(1-p_0)) \right] \\
&+ \frac{C(1-d_r)}{T - \tau} \left[\frac{1}{T - \tau} \left(-\frac{\alpha\beta\tau}{\theta_u} + \frac{pp_0}{\theta_u} + \frac{\alpha\beta}{\theta_u^2} + \frac{\alpha}{\theta_u} - \frac{p}{\theta_u} \right) \right. \\
&\quad \left. - \frac{\alpha\beta}{\gamma} e^{\theta_u(T-\tau)} + e^{\theta_u(T-\tau)} \left(\frac{\alpha\beta T}{\theta_u} - \frac{pp_0}{\theta_u} - \frac{\alpha\beta}{\theta_u^2} - \frac{\alpha}{\theta_u} + \frac{p}{\theta_u} \right) \left(\frac{1}{T - \tau} - \theta_u \right) \right] \\
&- \frac{h_u}{2T^2\theta_u^3} \left[-\alpha\beta\theta_u^2(T^2 - \tau^2) + 2pp_0(T - \tau) + 2\alpha\beta\theta_u T e^{\theta_u(T-\tau)} \right. \\
&\quad + 2T\alpha\theta_u^2 - 2p\theta_u^2(T - \tau) - 2\alpha\beta\tau\theta_u - 2\alpha\tau\theta_u^2 \\
&\quad \left. + 2pp_0\theta_u(1 - e^{-\theta_u\tau}) - 2\alpha e^{-\theta_u\tau}(\theta_u + \beta) + 2e^{-\theta_u\tau}p\theta_u + 2\alpha(\beta - \theta_u) - 2p\theta_u \right] \\
&+ \frac{h_u}{2T\theta_u^3} \left[\theta_u^3\alpha\beta(T^2 - \tau^2) - 2\alpha\beta\theta_u^2(T - \tau) - 2\theta_u^3pp_0(T - \tau) \right. \\
&\quad \left. + 2\alpha\beta\theta_u(e^{(T-\tau)} - 1) + 2\theta_u^3T(p - \alpha) - 2\theta_u^3\tau(p - \alpha) \right] \\
&+ \frac{h_u}{2T\theta_u^2} \left[-\theta_u^2\alpha\beta(T^2 - \tau^2) + 2\theta_u^2pp_0(T - \tau) + 2\alpha\beta\theta_u(Te^{(T-\tau)} - \tau) \right. \\
&\quad - 2T\theta_u^2(p - \alpha) + 2\tau\theta_u^2(p - \alpha) + 2pp_0\theta_u(1 - e^{-\theta_u\tau}) \\
&\quad \left. + 2\alpha\beta(1 - e^{-\theta_u\tau}) + 2\alpha\theta_u(1 - e^{-\theta_u\tau}) - p\theta_u(1 - e^{-\theta_u\tau}) \right] = 0
\end{aligned} \tag{3.2.19}$$

Theorem 3.2.2: *Given the fixed positive p with $p > C$, then*

(a) *Equation (3.2.19) has one and only one solution.*

(b) *The sufficient conditions for maxima satisfied by the value of T obtained in (a).*

Proof: To show second order derivative of $TP(p, T)$ with respect to T is less than zero.

$$\begin{aligned}
\frac{\partial^2 TP(p, T)}{\partial T^2} &= -\frac{2Cap^{-b}}{T} \psi_1 - \frac{hap^{-b}e^{-T\varepsilon}}{T(\varepsilon - \theta)\theta} \psi_2 - \frac{hap^{-b}e^{-T\varepsilon}}{T(\varepsilon - \theta)\theta\varepsilon} \psi_3 - \left(\frac{2A}{T^3} - \eta_1 \right) \\
&- \frac{h_u}{T\theta_u^2} \psi_4 - \left(\frac{h_u}{2T\theta_u} + \frac{h_u}{T^3\theta_u^3} \right) \psi_5 - \frac{h_u}{2T\theta_u^3} \psi_6 + \left\{ \left(-\frac{h_u}{T^2\theta_u^2} \psi_7 - \frac{h_u}{T^2\theta_u^3} \psi_8 \right) + \eta_2 \right\} \\
&- \frac{ap^{-b+1}}{T} \psi_9 - \frac{C(1-d_r)}{(T - \tau)} \psi_{10}
\end{aligned} \tag{3.2.20}$$

Since, (3.2.20) satisfied following conditions,

$$a > 0, 0 < b < 1, 0 < \varepsilon < 1, 0 < \beta < 1, p \geq 0, T > \tau, p > C \text{ and } \varepsilon \neq \theta, 0 < \gamma < 1$$

and

$$\psi_1 = \frac{1}{T^2(\varepsilon - \theta)} - e^{T(\varepsilon - \theta)} \left(\frac{(\varepsilon - \theta)}{2} + \frac{1}{T} + \frac{1}{T^2(\varepsilon - \theta)} \right) > 0,$$

$$\psi_2 = (e^{T\varepsilon}\theta - e^{T\theta}\varepsilon + \varepsilon - \theta) \left(\varepsilon + \frac{2}{T} + \frac{1}{T^2\varepsilon} \right) > 0, \psi_3 = \theta\varepsilon^2 e^{T\varepsilon} - \theta^2 \varepsilon e^{T\theta} > 0,$$

$$\psi_4 = \left[\begin{array}{l} -\theta_u^3 \alpha \beta (T^2 - \tau^2) + 2\alpha \beta \theta_u^2 (T - \tau) + 2\theta_u^3 p p_0 (T - \tau) \\ -2\theta_u^3 (p - \alpha)(T - \tau) + 2\alpha \beta \theta_u (e^{\theta_u(T - \tau)} - 1) \end{array} \right] > 0,$$

$$\psi_5 = \left[\begin{array}{l} \theta_u^2 \alpha \beta (T^2 - \tau^2) - 2\theta_u^2 p p_0 (T - \tau) + 2\theta_u^2 (p - \alpha)(T - \tau) \\ + 2\alpha \beta \theta_u (\tau - T e^{\theta_u(T - \tau)}) + 2(e^{\theta_u(T - \tau)} - 1)(\theta_u p p_0 + \alpha \beta + \alpha \theta_u - p \theta_u) \end{array} \right] > 0,$$

$$\psi_6 = \left[\begin{array}{l} \theta_u^4 \alpha \beta (T^2 - \tau^2) - 2\theta_u^4 p p_0 (T - \tau) - 2\theta_u^3 \alpha \beta (2T - \tau) \\ + 2\theta_u^4 (p - \alpha)(T - \tau) + 2\theta_u^3 (\alpha - p) + 2\theta_u^3 p p_0 \end{array} \right] > 0,$$

$$\psi_7 = \left[\begin{array}{l} -\theta_u^2 \alpha \beta (T^2 - \tau^2) + 2\theta_u^2 p p_0 (T - \tau) + 2\alpha \beta \theta_u (T e^{\theta_u(T - \tau)} - \tau) \\ + 2\theta_u^2 (\alpha - p)(T - \tau) + 2(1 - e^{\theta_u(T - \tau)})(\theta_u p p_0 + \alpha \beta + \alpha \theta_u - p \theta_u) \end{array} \right] < 0,$$

$$\psi_8 = \left[\begin{array}{l} \theta_u^3 \alpha \beta (T^2 - \tau^2) - 2\theta_u^3 p p_0 (T - \tau) - 2\alpha \beta \theta_u^2 (T - \tau) \\ + 2\theta_u^3 (p - \alpha)(T - \tau) + 2\alpha \beta \theta_u (e^{\theta_u(T - \tau)} - 1) \end{array} \right] < 0,$$

$$\psi_9 = \left[\left(\varepsilon + \frac{2}{T} + \frac{2}{T^2\varepsilon} \right) e^{-T\varepsilon} - \frac{2}{T^2\varepsilon} + \alpha \beta p(1 - p_0) \right] > 0,$$

$$\psi_{10} = \left[\begin{array}{l} \frac{2}{(T - \tau)^2} \left(\left(\frac{-\alpha \beta \tau}{\theta_u} + \frac{\alpha \beta}{\theta_u^2} + \frac{p p_0}{\theta_u} + \frac{\alpha}{\theta_u} - \frac{p}{\theta_u} \right) + e^{\theta_u(T - \tau)} \left(\frac{\alpha \beta T}{\theta_u} - \frac{\alpha \beta}{\theta_u^2} - \frac{p p_0}{\theta_u} - \frac{\alpha}{\theta_u} + \frac{p}{\theta_u} \right) \right) \\ + e^{\theta_u(T - \tau)} \theta_u^2 \left(\frac{2\alpha \beta}{\theta_u^2} + \frac{\alpha \beta T}{\theta_u} - \frac{\alpha \beta}{\theta_u^2} - \frac{p p_0}{\theta_u} - \frac{\alpha}{\theta_u} + \frac{p}{\theta_u} \right) \\ - \frac{2}{(T - \tau)} \left(\left(\frac{\alpha \beta}{\theta_u} \right) e^{\theta_u(T - \tau)} + e^{\theta_u(T - \tau)} \theta_u \left(\frac{\alpha \beta T}{\theta_u} - \frac{\alpha \beta}{\theta_u^2} - \frac{p p_0}{\theta_u} - \frac{\alpha}{\theta_u} + \frac{p}{\theta_u} \right) \right) \end{array} \right] > 0,$$

$$\eta_1 = \frac{2h a p^{-b} e^{-T\varepsilon}}{T(\varepsilon - \theta)\theta} \left((e^{T\varepsilon}\varepsilon\theta - e^{T\theta}\theta\varepsilon) \left(1 + \frac{1}{T\varepsilon} \right) \right) \text{ and } \frac{2A}{T^3} > \eta_1,$$

$$\eta_2 = \frac{2p(1 - p_0)}{T} \left[\frac{\left(\frac{-\alpha \beta}{2} (T^2 - \tau^2) + \alpha(T - \tau) - p(1 - p_0)(T - \tau) \right)}{T^2} \right. \\ \left. - \frac{(-\alpha \beta T + \alpha - p(1 - p_0))}{T} \right] < 0 \text{ and}$$

$$\left(-\frac{h_u}{T^2\theta_u^2}\psi_7 - \frac{h_u}{T^2\theta_u^3}\psi_8\right) + \eta_2 < 0 ; \text{ it is shows that,}$$

$$\begin{aligned} \frac{\partial^2 TP(p,T)}{\partial T^2} = & -\frac{2Cap^{-b}}{T}\psi_1 - \frac{hap^{-b}e^{-T\varepsilon}}{T(\varepsilon-\theta)\theta}\psi_2 - \frac{hap^{-b}e^{-T\varepsilon}}{T(\varepsilon-\theta)\theta\varepsilon}\psi_3 - \left(\frac{2A}{T^3} - \eta_1\right) \\ & - \frac{h_u}{T\theta_u^2}\psi_4 - \left(\frac{h_u}{2T\theta_u} + \frac{h_u}{T^3\theta_u^3}\right)\psi_5 - \frac{h_u}{2T\theta_u^3}\psi_6 + \left\{\left(-\frac{h_u}{T^2\theta_u^2}\psi_7 - \frac{h_u}{T^2\theta_u^3}\psi_8\right) + \eta_2\right\} \\ & - \frac{ap^{-b+1}}{T}\psi_9 - \frac{C(1-d)}{(T-\tau)}\psi_{10} < 0 \end{aligned} \quad (3.2.21)$$

Hence, (3.2.19) has one and only one solution and the sufficient condition for maxima satisfied as in (3.2.21) by the optimal value of selling price.

3.2.3 Numerical experiment

The proposed models are illustrated below by considering the following example.

Example 3.2.1: The numerical values of the parameter in proper unit were considered as input for numerical, graphical and sensitivity analysis of the model.

The scale demand of new product $a = 255$ units, price elasticity of new product $b = 0.4$, $\varepsilon = 0.9$, $\alpha = 100$, $\beta = 0.3$, purchasing cost $C = ₹55$ per unit, ordering cost $A = ₹100$ per order, holding cost of new product $h = ₹0.5/\text{unit}/\text{year}$, holding cost of used buyback product $h_u = ₹0.2/\text{unit}/\text{year}$, rate of depreciation of buyback product $d_r = 0.15$, $\tau = \frac{30}{365}$ year, price discount on selling price of used buyback product $p_0 = 0.5$, rate of deterioration of new product and used buyback product $\theta = 0.01$ and $\theta_u = 0.02$ respectively.

Using mathematical software like maple 18 or MATLAB or Mathematica, the optimal results of proposed model given in Table 3.3, and validation of sufficient conditions by hessian matrix method are given below:

Table 3.3 Optimal results of model 3.2

p^* (in ₹)	T^* (year)	Q^* (units)	Q_u^* (units)	Total Profit (in ₹)
103.7220	0.37085	12.58	11.97	4980.21

The concavity of the profit function is developed by the well-known Hessian matrix, Consider Hessian Matrix as following,

$$H(p,T) = \begin{pmatrix} \frac{\partial^2 TP(p,T)}{\partial p^2} & \frac{\partial^2 TP(p,T)}{\partial p \partial T} \\ \frac{\partial^2 TP(p,T)}{\partial T \partial p} & \frac{\partial^2 TP(p,T)}{\partial T^2} \end{pmatrix} \quad (3.2.22)$$

$$H(p^*, T^*) = \begin{pmatrix} -0.5648122516 & -20.48479785 \\ -20.48479785 & -11351.81445 \end{pmatrix} \quad (3.2.23)$$

As per Cárdenas-Barrón and Sana[203], If the Eigen values of the Hessian matrix at the solution (p^*, T^*) are all negative then the profit function $TP(p^*, T^*)$ is maximum at the solution. Here, eigenvalues of the Hessian matrix (3.2.23) are $\lambda_1 = -11351.85 < 0$ and $\lambda_2 = -0.53 < 0$. Therefore, the profit function $TP(p^*, T^*)$ is maximum.

From above Hessian Matrix, define that $\Delta_{11} = \frac{\partial^2 \pi(p,T)}{\partial p^2}$, $\Delta_{22} = \frac{\partial^2 \pi(p,T)}{\partial T^2}$ and

$\Delta_{12} = \frac{\partial^2 \pi(p,T)}{\partial p \partial T}$ for optimal value of p^* and T^* , it is clear that $\Delta_{11} = -0.56 < 0$,

$\Delta_{22} = -11351.81 < 0$ and $\Delta_{11}\Delta_{22} - (\Delta_{12})^2 > 0$ then the optimal value of p^* and T^* satisfies the (3.2.17) and (3.2.19) and value of p^* and T^* is unique and maximize $\pi(p,T)$.

3.2.3.1 Graphical authentication of the concavity of objective function

The concavity illustrations of the objective function of proposed model 3.2 are presented in Figure 3.4, Figure 3.5 and Figure 3.6.

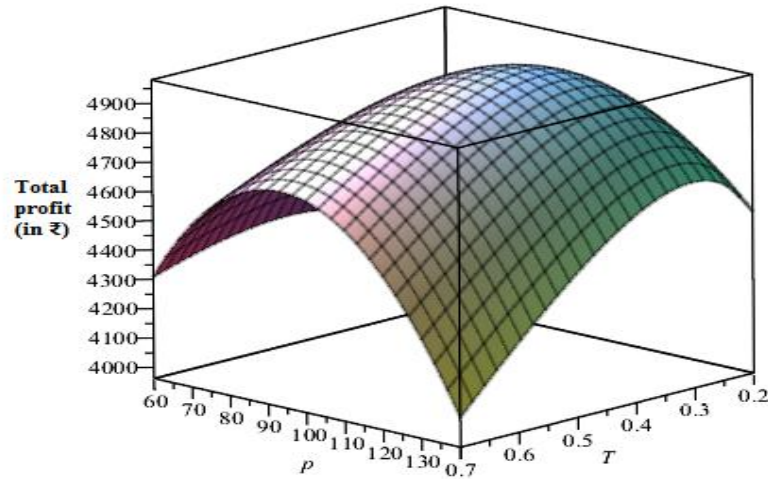


Figure 3.4 Concavity of total profit function with respect to p and T for model 3.2

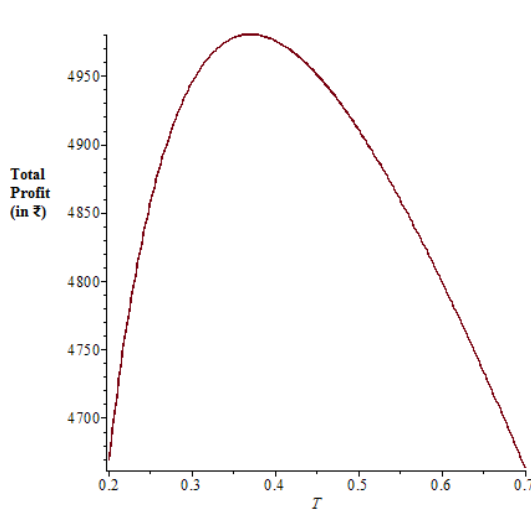


Figure 3.5 Total Profit vs Cycle time for model 3.2

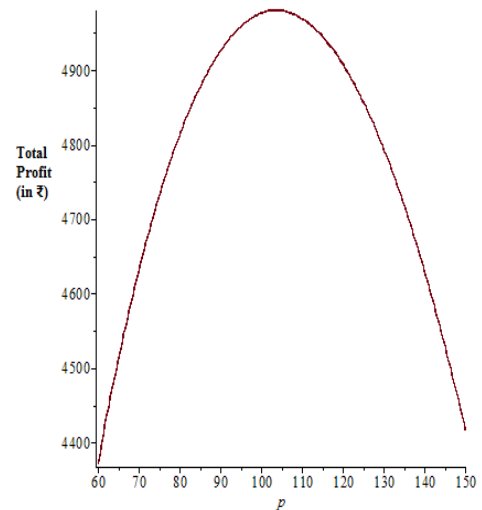


Figure 3.6 Total Profit vs Selling Price for model 3.2

3.2.4 Sensitivity Analysis

Using the mathematical software Maple 18 or Matlab or mathematica, Table 3.4 revealed the variation in optimal outcomes resulting from the sensitivity of system parameters between -20% and +20%. When determining the sensitivity of a parameter of example 3.2.1, one parameter is taken at a time while the other parameter's values remain unaltered.

Table 3.4 Sensitivity with respect to key parameters

Inventory Parameter	Change %	Value	T^*	p^*	Q^*	Q_u^*	Profit (in ₹)	Eigen Values of (3.2.19) (λ_1, λ_2)
a	-20	204	0.4038	92.68	11.31	14.96	4684.57	(-8845.88,-0.54)
	-10	229.5	0.3873	98.23	12.00	13.42	4824.05	(-10015.90,-0.54)
	0	255	0.3708	103.72	12.58	11.97	4980.21	(-11351.85,-0.53)
	10	280.5	0.3546	109.19	13.05	10.61	5152.64	(-12874.72,-0.52)
	20	306	0.3387	114.66	12.31	9.35	5144.76	(-14610.24,-0.52)
b	-20	0.32	0.2860	135.16	13.39	5.49	5947.61	(-22413.16,-0.41)
	-10	0.36	0.3340	116.86	13.27	8.92	5359.55	(-16870.10,-0.48)
	0	0.4	0.3708	103.72	12.58	11.97	4980.21	(-11351.85,-0.53)
	10	0.44	0.3998	93.69	11.64	14.63	4729.78	(-9233.21,-0.55)
	20	0.48	0.4225	85.76	10.60	16.92	4562.41	(-7687.54,-0.58)
α	-20	80	0.3672	91.52	13.12	8.24	3399.65	(-8763.60,-0.55)
	-10	90	0.3686	97.51	10.10	10.10	4178.73	(-10057.40,-0.54)
	0	100	0.3708	103.72	12.58	11.97	4980.21	(-11351.85,-0.53)
	10	110	0.3737	110.16	12.36	13.86	5805.05	(12852.29,-0.54)
	20	120	0.3770	116.80	12.16	15.77	6654.21	(-14012.40,-0.50)
β	-20	0.24	0.4012	103.64	13.44	7.16	5103.21	(-8918.77,-0.53)
	-10	0.27	0.3851	103.69	12.99	12.71	5040.58	(-10143.23,-0.53)
	0	0.3	0.3708	103.72	12.58	11.97	4980.21	(-11351.85,-0.53)
	10	0.33	0.3581	103.74	12.21	11.30	4921.89	(-12690.10,-0.52)
	20	0.36	0.3466	103.76	11.88	10.71	4865.44	(-14110.52,-0.51)
C	-20	44	0.3574	110.33	11.90	10.55	4970.51	(-12486.98,-0.48)
	-10	49.5	0.3642	106.93	12.24	11.27	4975.59	(-11918.24,-0.51)
	0	55	0.3708	103.72	12.58	11.97	4980.21	(-11351.85,-0.53)
	10	60.5	0.3774	100.69	12.92	12.66	4989.97	(-10830.21,-0.55)
	20	66	0.3839	97.83	13.26	13.34	5004.50	(-10347.50,-0.57)
A	-20	80	0.3569	104.24	12.15	11.37	5035.18	(-11892.53,-0.54)
	-10	90	0.3639	103.98	12.37	11.67	5007.4	(-11513.12,-0.53)
	0	100	0.3708	103.72	12.58	11.97	4980.21	(-11351.85,-0.53)
	10	110	0.3777	103.47	12.79	12.26	4953.50	(10998.11,-0.52)
	20	120	0.3844	103.23	12.99	12.54	4927.23	(-10866.13,-0.51)
d_r	-20	0.12	0.3714	102.24	12.67	12.20	5049.23	(-11319.50,-0.53)
	-10	0.135	0.3711	102.98	12.62	12.09	5014.55	(-11335.07,-0.53)
	0	0.15	0.3708	103.72	12.58	11.97	4980.21	(-11351.85,-0.53)
	10	0.165	0.3705	104.47	12.54	11.85	4946.16	(-11366.19,-0.53)
	20	0.18	0.3702	105.22	12.49	11.73	4912.41	(-11387.14,-0.53)
h	-20	0.4	0.3710	103.71	12.58	11.97	4980.81	(-11341.60,-0.53)
	-10	0.45	0.3709	103.72	12.58	11.97	4980.49	(-11346.21,-0.53)
	0	0.5	0.3708	103.72	12.58	11.97	4980.21	(-11351.85,-0.53)
	10	0.55	0.3708	103.73	12.58	11.96	4979.91	(-11355.92,-0.53)
	20	0.6	0.3707	103.73	12.58	11.96	4979.63	(-11361.80,-0.53)
h_u	-20	0.16	0.3709	103.72	12.58	11.97	4980.39	(-11345.60,-0.53)
	-10	0.18	0.3709	103.72	12.58	11.97	4980.28	(-11348.23,-0.53)
	0	0.2	0.3708	103.72	12.58	11.97	4980.21	(-11351.85,-0.53)

Inventory Parameter	Change %	Value	T^*	p^*	Q^*	Q_u^*	Profit (in ₹)	Eigen Values of (3.2.19) (λ_1, λ_2)
	10	0.22	0.3708	103.73	12.58	11.97	4980.12	(-11354.11,-0.53)
	20	0.24	0.3708	103.73	12.58	11.96	4980.05	(-11357.70,-0.53)
τ	-20	0.0658	0.3437	104.46	11.76	11.60	5101.48	(-12416.54,-0.54)
	-10	0.0740	0.3575	104.09	12.18	11.79	5039.67	(-11883.50,-0.53)
	0	0.0822	0.3708	103.72	12.58	11.97	4980.21	(-11351.85,-0.53)
	10	0.0904	0.3838	103.36	12.97	12.12	4922.86	(-10932.02,-0.52)
	20	0.0986	0.3964	103.01	13.34	12.26	4867.43	(-10485.44,-0.52)
p_0	-20	0.4	0.3986	83.38	14.58	13.57	4480.58	(-9232.77,-0.78)
	-10	0.45	0.3851	92.48	13.59	12.80	4714.00	(-10189.88,-0.65)
	0	0.5	0.3708	103.72	12.58	11.97	4980.21	(-11351.85,-0.53)
	10	0.55	0.3557	117.94	11.54	11.07	5288.79	(12988.38,-0.50)
	20	0.6	0.3393	136.43	10.45	10.08	5653.79	(-14638.19,-0.47)
ε	-20	0.72	0.3789	105.01	13.17	12.07	5034.12	(-10737.55,-0.53)
	-10	0.81	0.3747	104.36	12.86	12.02	5006.61	(-11058.12,-0.53)
	0	0.9	0.3708	103.72	12.58	11.97	4980.21	(-11351.85,-0.53)
	10	0.99	0.3674	103.09	12.32	11.93	4954.86	(-11618.79,-0.53)
	20	1.08	0.3643	102.47	12.07	11.90	4930.46	(-11860.58,-0.53)
θ	-20	0.008	0.3710	103.71	12.58	11.97	4980.88	(-11340.13,-0.53)
	-10	0.009	0.3709	103.72	12.58	11.97	4980.53	(-11345.75,-0.53)
	0	0.01	0.3708	103.72	12.58	11.97	4980.21	(-11351.85,-0.53)
	10	0.011	0.3708	103.73	12.58	11.96	4979.90	(-11357.29,-0.53)
	20	0.012	0.3707	103.73	12.58	11.96	4979.54	(-11364.12,-0.53)
θ_u	-20	0.016	0.3705	103.76	12.57	11.94	4979.12	(-11380.87,-0.53)
	-10	0.018	0.3707	103.74	12.57	11.95	4979.66	(-11366.13,-0.53)
	0	0.02	0.3708	103.72	12.58	11.97	4980.21	(-11351.85,-0.53)
	10	0.022	0.3710	103.70	12.59	11.98	4980.76	(-11337.29,-0.53)
	20	0.024	0.3712	103.69	12.59	11.99	4981.30	(-11322.77,-0.53)

In the sensitivity analysis obtained in Table 3.4, the following observations were made:

- Here, it is observed that the eigen-values of the hessian matrix at the corresponding value of p^* and T^* all are negative, meaning that profit is maximized at (p^*, T^*) .
- The scale demand a and parameter α have positive impact on selling price and total profit of retailer.
- An increasing the ordering cost A lead to gradually decrease the selling price and increases the cycle time, while the total profit will be decreases.
- If the increases in h , h_u then slightly increases selling price, decreases cycle time and profit. Ordering quantity and buyback quantity slightly decreases due to increase in h , h_u .

- If the increases in d_r implies that profit, ordering quantity, buyback quantity and cycle time decreases. On other side, selling price will be slightly decreases.
- Selling price discount p_0 facility on used buyback product is more effective to gain the retailer's total profit. Increases in p_0 result to increases the selling price and profit while decreases the cycle time.
- Optimal selling price increase when system parameters a, α, d_r, h, h_u and p_0 increases but if parameters $b, C, \varepsilon, \beta, A, \tau$ increase then selling price decrease. Admittedly, selling price is highly positive sensitive to a, α, d_r, p_0 and strongly negative sensitive to b, C, ε .
- When the value of the parameters a, α, p_0 increase, the optimal total profit will be increase, However, for increasing in parameter $b, \beta, C, A, d_r, \varepsilon, h, \tau, \theta$ then total profit will decrease.
- It is noted that replenishment cycle time T is positively related to system parameters b, A, τ and negatively related to a, α, β . However, not much effect in cycle time for change in holding cost parameters and remaining others.
- Increases in θ , total profit of the retailer is gradually decreases, order quantity and cycle time decreases and selling price decreases. Retailer's total profit may gradually increases (if $\theta_u \geq \theta$) due to increase the rate of deterioration θ_u of used buyback product. This finding implies that the higher selling price and higher rate of deterioration of new product may negative effects on total profit but lower selling price, higher ordering quantity and higher rate of deterioration of used product compare to new products may positive impacts on retailer's total profit.

3.3 Analysis of deterioration effect's on retailer's profit

The rate of deterioration of the new and used products is how it affects the retailer's profit and how it affects the ordering quantity and the buyback quantity, which are derived numerically in Table 3.5:

Table 3.5 Effect of products deterioration on retailer's profit

Case(s):	θ	θ_u	Q^* (units)	Q_u^* (units)	Total Profit (in ₹)	Profit behaviour
Rate of Deterioration is zero. (Model 3.1)	0	0	12.52	11.88	4978.10	
Rate of deterioration is same for both products	0.01	0.01	12.55	11.90	4977.53	↓
Rate of deterioration of used buyback product is higher than new products	0.01	0.012	12.56	11.92	4978.06	
Rate of deterioration of used buyback products is double than new products (Model 3.2)	0.01	0.02	12.58	11.97	4980.21	↑
Rate of deterioration of new products is higher than used products.	0.01	0.008	12.54	11.89	4976.98	
	0.01	0.005	12.53	11.97	4976.23	↓

3.4 Discussion about managerial insights

According to the behavioural changes revealed by the sensitivity analysis and mathematical modelling, the following managerial insights might be drawn:

- A higher scale demand of new products inspires a retailer to set a high selling price and gain more profit. The constant return rate of used products and corresponding demand for used products improve the retailer's profit, and it is also beneficial to environmental protection because higher demand for used products reduces the need for raw materials for the manufacturing process of new products.
- The higher ordering cost is negatively proportional to profit. This finding implies that the ordering cost should be properly maintained by the retailer to increase their profit.
- Higher value of holding cost for new product and used product which negative impacts on retailer's total profit. So, retailer should try to reduce holding cost for new products and used products for reduce the loss.
- Our analysis shows that retailers who give a higher price discount on used products during resale to customers increase total profit by increasing the selling price.
- The higher rate of deterioration of new product which affects gradually decreases the retailer's total profit.

- In the case of non-deteriorating products, the retailer's profit is higher compared to considering deteriorating products. It is clear that the products with deterioration rates comparable to zero have low preservation and deterioration costs, which will help to increase the profit.
- Natural phenomena make it clear that old products deteriorate at a faster rate than newer ones. Our analysis demonstrates that the retailer's profit rises if the rate of deterioration for used products is higher than for new products. If we assume that the rate of deterioration of old products is lower than that of new products, our study indicates profit falls. The rate of deterioration of old products is lower than new products, which is not always possible.
- Higher rate of depreciation on purchase cost for used buy back product which has a negative impact on retailer's total profit.

3.5 Conclusion

In this chapter, two types of inventory models are established as retailer-centric decisions for products with and without deterioration impacts. People are willing to purchase a used item from a retailer in the modern marketplace since it has the same features as a new product, is less expensive, and is environmentally conscious, in addition to being interested in purchasing a newly made or just released product. In order to maximize the retailer's total profit, this effort aims to develop an inventory system that both sells new products and recovers used products from customers to resale. We presented a mathematical formulation of an inventory system with and without deterioration, and the optimal selling price, replenishment time, ordering quantity of new product, and optimal buyback quantity of used product are determined using classical optimization. Consumer demand and pricing are closely linked. When market prices rise, market demand declines, and inversely. The novel points of this chapter are that we postulate a demand is price-sensitive, time-dependent, and exponentially declining because time has an impact on demand, and the different cases of the rate of deterioration of new products and old buyback products, as well as how they impact the retailer's profit and the optimal quantity of products, are discussed. The proposed models have been supported by numerical examples, and the variation effects of parameters on optimal results are evaluated through sensitivity analysis. The managerial insights deduced from the analysis as outcomes of proposed study. 'A simple example of this model is supposed to be taken from a military point of view,

afterward shooting practice, the bullet coats have been recovered, and obeying the cleanup process, they can be used as new bullet coats. In this situation, the size of the buyback quantity and the order quantity for replacement coats should be decided simultaneously'. A possible future study as an extension of this chapter may consider the rework of used buyback products and again selling them, stock-dependent demand, advertisement-dependent demand, trade credit policy, shortages, etc., as the case may be.

CHAPTER-4

Optimal Pricing and Replenishment Strategies for New Products and Buyback strategy of Used Products from the Retailer's Points under Partial Backlog Shortages

4.0 Introduction

World human civilization has a genuine interest in sustainability issues. The consumer demand for sustainable production and reuse is growing. In the current market environment, consumers aren't solely interested in purchasing recently manufactured or released products; they're also excited to purchase a used product from a merchant because it offers the same characteristics as new products at a lower price. In these circumstances, retrieving used products is cheaper than discarding them. It's not a recent occurrence for people to reuse objects and resources. In the last two to three decades, the process of recycling or refurbishing products has been commonplace for products including paper, metal, glass, and jewellery etc. Currently, it has been noticed that plastic bags, water bottles, cell phones, marker pens, and other items are all recycled or reused in the same way. In our analysis, we looked for reusable goods with a simple buyback process. The stock out situations may not be avoided during the management of inventory system. Other factor is product deterioration play the major role in inventory decisions.

Keep all these points in mind. In this chapter, we developed two inventory models for the cases without deterioration and with deterioration, in which the retailer is the decision maker with shortages allowed and demand fulfilled by new products as well as buyback products and the unsatisfied demand is partially backlogged. The contributions of this chapter are: (i) the retailer sell the new product to the customers as well as collect the used

product during the inventory period; the collected used products are to be sold to the customers during cycle time. (ii) Demand is price-sensitive and exponentially declines with time for new products and linearly decreases with price and time for used products. (iii) to boost the demand for used buyback products, retailers give a price discount on the selling price of used buyback products to the customer; (iv) the rate of deterioration is to be considered constant for new as well as used buyback products; and the impact of deterioration on the retailer's profit is identified. With regard to the optimum selling price, positive inventory period, shortages period and ordering quantity for new products as well as the optimum buyback quantity for used products, the aim is to optimize total profit for the retailer. Global optimality of the objective function is verified using the hessian matrix method and a graphical representation. The management implications for the best feasible opportunity in terms of parameters have been emphasized through sensitivity analysis. Additionally, some concluding observations and suggestions for further research are given. In this chapter, two inventory models are developed, as follows:

- Model 4.1 Optimal inventory strategy for non-deteriorating products for which shortages are partially backlogged
- Model 4.2 Optimal inventory strategy for deteriorating products for which shortages are partially backlogged

4.1 Optimal inventory strategy for non-deteriorating products for which shortages are partially backlogged

In this proposed model, we assume that the retailer sells products that are non-deteriorating during the cycle time. The demand for the products is price and time-dependent which is satisfied by both types of products and unsatisfied demand is partially backlogged.

4.1.1 Notations and Assumptions

4.1.1.1 Notations

Parameters

A Retailer's ordering cost (in ₹/order).

C	Purchase cost (constant) (in ₹/unit).
h	Inventory holding cost (in ₹/unit) for new product.
h_u	Inventory holding cost (in ₹/unit) for used buy back product.
τ	The point of time when collection and sell of used buy back products starts (years), $0 \leq \tau \leq t_1$.
p_0	Rate of discount on selling price for used buy back product.
d_r	Rate of depreciation on purchase cost for used buyback product.
b_1	Backordered cost per unit per unit time for new product.
b_2	Backordered cost per unit per unit time for used buyback product.
l_1	Cost of lost sales per unit per unit time for new product.
l_2	Cost of lost sales per unit per unit time for used buyback product.
δ	Backlogging parameter, $0 < \delta < 1$

Decision variables

t_1	Time at which the inventory level reaches zero (a decision variable) (Year).
t_2	Length of the period during which shortages are allowed. (a decision variable) (Year).
p	Selling Price (in ₹/unit) (a decision variable).

Objective function

$TP(t_1, t_2, p)$	Total profit function of the retailer which is the sum of profit generated from new products and buy-back used products (in ₹).
-------------------	---

Other expressions and functions

$R_n(p, t)$	Demand rate for new product at $0 \leq t \leq t_1$ (units).
$R_u(p, t)$	Demand rate for used buyback product at $\tau \leq t \leq t_1$ (units).
$I_1(t)$	Inventory level at time $0 \leq t \leq t_1$ for new product (units).
$I_2(t)$	Inventory level at time $0 \leq t \leq t_2$ for new product (units).
$I_{u1}(t)$	Inventory level at time $0 \leq \tau \leq t_1$ for used product (units).
$I_{u2}(t)$	Inventory level at time $0 \leq t \leq t_2$ for used product (units).
IM	Maximum inventory level for new product during $0 \leq t \leq t_1$.
IB	Maximum backordered units of new product during stock out period.

IM_u	Maximum inventory level for used buyback product during $0 \leq \tau \leq t \leq t_1$.
IB_u	Maximum backordered units for used buyback product during stock out period.
Q	The replenishment quantity for new product.
Q_u	The quantity of used buy back product.
$TP_n(t_1, t_2, p)$	Profit generated from new products (in ₹).
$TP_u(t_1, t_2, p)$	Profit generated from buy-back used products (in ₹).

4.1.1.2 Assumptions

1. The inventory system comprises only single type of product.
2. The replenishment is instantaneous and planning horizon is infinite.
3. The holding cost is considered to be constant for new product as well as used buyback product and $h > h_u$.
4. The rate of demand for new product is taken as a

$$R_n(p, t) = \begin{cases} ap^{-b}e^{-\varepsilon t}; & 0 \leq t \leq t_1 \\ ap^{-b}; & 0 \leq t \leq t_2 \end{cases}, t_1 + t_2 = T$$

where, $a > 0$ denotes the scale demand, $0 < b < 1$ denotes the price elasticity and $0 < \varepsilon < 1$ is time sensitive demand parameter.

5. The buyback rate of used product is,

$$R_u(p, t) = \begin{cases} \alpha(1 - \beta t) - p(1 - p_0); & \tau \leq t \leq t_1 \\ \alpha - p(1 - p_0); & 0 \leq t \leq t_2 \end{cases}$$

where, $\alpha > 0$ denotes the scale demand and $0 < \beta < 1$ is time sensitive demand parameter.

6. The Lead time is negligible or zero.

7. A retailer sells the new product during $0 \leq t \leq t_1$ and buyback the used product at time τ and start the selling the used product during $\tau \leq t \leq t_1$.
8. A retailer sells the new product to customers as well as collects and sells the used products again. Rework or repairing of used buyback product is not considered.
9. Stock-out situations are permitted, and it is partially backlogged. The backlog rate varies during the stock-out duration and is based on how long it takes for the subsequent replenishment. The backlogging rate $e^{-\delta(x)}$ is a function of waiting period, where x is waiting period up to next delivery, $0 < \delta < 1$. (Abad[33])

4.1.2 Mathematical Formulation

The stock level of new product at the beginning of cycle time is maximum and retailer's inventory level decline during the cycle time $[0, t_1]$ due to the market demand, so status of new products stock at time t over the period $[0, t_1]$ can be represented by differential equation,

$$\frac{dI_1(t)}{dt} = -R_n(p, t), 0 \leq t \leq t_1 \quad (4.1.1)$$

At the end of cycle time the inventory level goes zero, so the solution of (4.1.1) at the boundary condition $I_1(t_1) = 0$ is,

$$I_1(t) = \frac{ap^{-b}}{\varepsilon} (e^{-\varepsilon t} - e^{-\varepsilon t_1}), 0 \leq t \leq t_1 \quad (4.1.2)$$

At time t_1 , the inventory level approached to zero and shortages occurred. Some buyers may be prepared to wait for a shipment delay during the stock-out time, while others may leave in search of another vendor due to an immediate need. For a customer who need to get the product during shortages period are wait for next delivery of products, the portion of consumers backordering can be written as $e^{-\delta(x)}$, where x is waiting period up to next delivery (Abad[33]). So the inventory level of new product during shortages period $[0, t_2]$ can be expressed by the differential equation,

$$\frac{dI_2(t)}{dt} = -R_n(p, t)e^{-\delta(t_2-t)}, 0 \leq t \leq t_2, I_2(0) = 0 \quad (4.1.3)$$

The solution of the differential equation (4.1.3) is,

$$I_2(t) = -\frac{ap^{-b}}{\delta} e^{-\delta t_2} (e^{\delta t} - 1) \quad (4.1.4)$$

The maximum positive inventory for new product is,

$$IM = I_1(0) = \frac{ap^{-b}}{\varepsilon} (1 - e^{-\varepsilon t_1}) \quad (4.1.5)$$

The maximum backordered units of new product are,

$$IB = -I_2(t_2) = \frac{ap^{-b}}{\delta} (1 - e^{-\delta t_2}) \quad (4.1.6)$$

Thus, the replenishment quantity of new products over the cycle time is derived as,

$$Q = IM + IB = \frac{ap^{-b}}{\varepsilon} (1 - e^{-\varepsilon t_1}) + \frac{ap^{-b}}{\delta} (1 - e^{-\delta t_2}) \quad (4.1.7)$$

Stock level of used products, which depends upon the buyback rate. As per the assumptions retailer start to collect the used product at time τ , and start the selling of used products from τ , inventory level of used product decreases due to demand and hence level of used buyback inventory $I_{u1}(t)$ during the period $[\tau, t_1]$, is presented through differential equation at any time $t \in [\tau, t_1]$ is,

$$\frac{dI_{u1}(t)}{dt} = -(\alpha(1 - \beta t) - p(1 - p_0)), 0 \leq \tau \leq t \leq t_1 \quad (4.1.8)$$

Applying the boundary condition $I_{u1}(t_1) = 0$, the solution of (4.1.8) is given by,

$$I_{u1}(t) = \frac{\alpha\beta}{2} (t^2 - t_1^2) + (p(1 - p_0) - \alpha)(t - t_1) \quad (4.1.9)$$

During the shortage period the inventory level depends on demand and a fraction $e^{-\delta(t_2-t)}$ of the demand is backlogged. The inventory level for used product is governed by the following differential equation,

$$\frac{dI_{u2}(t)}{dt} = -(\alpha - p(1 - p_0))e^{-\delta(t_2-t)}, 0 \leq t \leq t_2 \quad (4.1.10)$$

The solution of (4.1.10) is at $I_{u2}(0) = 0$ is given by,

$$I_{u2}(t) = -\frac{p(1-p_0)-\alpha}{\delta} e^{-\delta t_2} (1-e^{\delta t}), 0 \leq t \leq t_2 \quad (4.1.11)$$

The maximum inventory level of used product at time positive τ is,

$$IM_u = I_{u1}(\tau) = \frac{\alpha\beta}{2} (\tau^2 - t_1^2) + (p(1-p_0)-\alpha)(\tau - t_1) \quad (4.1.12)$$

The maximum backordered units of used product are,

$$IB_u = -I_{u2}(t_2) = \frac{p(1-p_0)-\alpha}{\delta} (e^{-\delta t_2} - 1), 0 \leq t \leq t_2 \quad (4.1.13)$$

Thus, buyback quantity of used product over the replenishment cycle can be derived as,

$$Q_u = \frac{1}{2} \alpha\beta(t_1^2 - \tau^2) + (p - pp_0 - \alpha)(t_1 - \tau) + \frac{p(1-p_0)-\alpha}{\delta} (1 - e^{-\delta t_2}) \quad (4.1.14)$$

To evaluate total profit of retailer from new product, we work out the following components:

Sales revenue generated from new product:

$$SR_n = \frac{p}{t_1 + t_2} \left(\int_0^{t_1} ap^{-b} e^{-\epsilon t} dt + \int_0^{t_2} ap^{-b} e^{-\delta(t_2-t)} dt \right) \quad (4.1.15)$$

Purchase cost for new product:

$$PC_n = \frac{CQ}{t_1 + t_2} \quad (4.1.16)$$

Cost for holding the new product:

$$HC_n = \frac{h}{t_1 + t_2} \int_0^{t_1} I_1(t) dt \quad (4.1.17)$$

New product's ordering cost:

$$OC_n = \frac{A}{t_1 + t_2} \quad (4.1.18)$$

Backordered cost due to shortages for new product:

$$BC_n = \frac{b_1}{t_1 + t_2} \int_0^{t_2} -I_2(t) dt \quad (4.1.19)$$

Lost sales cost during shortages for new product:

$$LS_n = \frac{l_1}{t_1 + t_2} \int_0^{t_2} ap^{-b}(1 - e^{-\delta(t_2-t)}) dt \quad (4.1.20)$$

Total profit of retailer's from new product earn from sales revenue minus all costs during the cycle is,

$$TP_n(t_1, t_2, p) = SR_n - OC_n - PC_n - HC_n - BC_n - LS_n \quad (4.1.21)$$

Now, to figure out the total profit of retailer from buyback used products, we enumerate the components as below:

Sales revenue from used buyback products:

$$SR_u = \frac{p(1-p_0)}{t_1 + t_2} \left(\int_{\tau}^{t_1} (\alpha(1-\beta t) - p(1-p_0)) dt + \int_0^{t_2} (\alpha - p(1-p_0)e^{-\delta(t_2-t)}) dt \right) \quad (4.1.22)$$

Used product's purchase cost:

$$PC_u = \frac{C(1-d_r)Q_u}{(t_1 + t_2) - \tau} \quad (4.1.23)$$

Holding cost for used buyback products:

$$HC_u = \frac{h_u}{t_1 + t_2} \int_{\tau}^{t_1} I_{u1}(t) dt \quad (4.1.24)$$

Backordered cost due to shortages for used products:

$$BC_u = \frac{b_2}{t_1 + t_2} \int_0^{t_2} -I_{u2}(t) dt \quad (4.1.25)$$

Lost sale cost for used products:

$$LS_u = \frac{l_2}{t_1 + t_2} \int_0^{t_2} (\alpha - p(1-p_0))(1 - e^{-\delta(t_2-t)}) dt \quad (4.1.26)$$

Total profit for used buyback product during the cycle is,

$$TP_u(t_1, t_2, p) = SR_u - PC_u - HC_u - BC_u - LS_u \quad (4.1.27)$$

Therefore, the total profit from the both products is given by from (4.1.21) and (4.1.27),

$$TP(t_1, t_2, p) = (SR_n - OC_n - PC_n - HC_n - BC_n - LS_n) + (SR_u - PC_u - HC_u - BC_u - LS_u) \quad (4.1.28)$$

$$\begin{aligned} TP(t_1, t_2, p) = & \frac{p}{t_1 + t_2} \left(\frac{ap^{-b}(1 - e^{-\delta t_2})}{\delta} + \frac{ap^{-b}(1 - e^{-\varepsilon t_1})}{\varepsilon} \right) - \frac{A}{t_1 + t_2} \\ & - \frac{Cap^{-b}}{t_1 + t_2} \left(\frac{1}{\varepsilon} - \frac{e^{-\varepsilon t_1}}{\varepsilon} + \frac{1}{\delta} - \frac{e^{-\delta t_2}}{\delta} \right) \\ & - \frac{hap^{-b}}{t_1 + t_2} \left(\frac{1 - e^{-\varepsilon t_1} - t_1 \varepsilon e^{-\varepsilon t_1}}{\varepsilon^2} \right) - \frac{b_1 ap^{-b}}{t_1 + t_2} \left(\frac{1 - e^{-\delta t_2} - t_2 \delta e^{\delta(t_1 - t_2)}}{\delta^2} \right) \\ & - \frac{l_1 ap^{-b}}{t_1 + t_2} \left(\frac{e^{-\delta t_2} + t_2 \delta - 1}{\delta} \right) \\ & + \frac{p(1 - p_0)}{t_1 + t_2} \left(\frac{-\frac{1}{2} \alpha \beta (t_1^2 - \tau^2) + \alpha(t_1 - \tau) - p(1 - p_0)(t_1 - \tau)}{+ \frac{(p(p_0 - 1) + \alpha)(1 - e^{-\delta t_2})}{\delta}} \right) \\ & - \frac{C(1 - d_r)}{t_1 + t_2 - \tau} \left(\frac{\frac{1}{2} \alpha \beta (t_1^2 - \tau^2) - (p(p_0 - 1) + \alpha)(t_1 - \tau)}{- \frac{(p(p_0 - 1) + \alpha)(1 - e^{-\delta t_2})}{\delta}} \right) \\ & - \frac{h_u}{t_1 + t_2} \left(\frac{\frac{1}{6} \alpha \beta (t_1^3 - \tau^3) - \frac{1}{2} (p(p_0 - 1) + \alpha)(t_1^2 - \tau^2)}{- \frac{1}{2} \alpha \beta t_1^2 (t_1 - \tau) + (p(p_0 - 1) + \alpha)t_1(t_1 - \tau)} \right) \\ & - \frac{b_2}{t_1 + t_2} \left(\frac{(p(p_0 - 1) + \alpha)(1 - \delta t_2 e^{\delta(t_1 - t_2)} - e^{-\delta t_2})}{\delta^2} \right) \\ & - \frac{l_2}{t_1 + t_2} \left(\frac{(p(p_0 - 1) + \alpha)(\delta t_2 + e^{-\delta t_2} - 1)}{\delta} \right) \end{aligned}$$

In above expression, to find the optimal value of optimal value of t_1, t_2 and p which maximize $TP(t_1, t_2, p)$.

4.1.2.1 Solution technique to determine the optimal solution

The necessary conditions for maximize the total profit function (4.1.28) are given by are,

$$\frac{\partial TP(t_1, t_2, p)}{\partial t_1} = 0, \frac{\partial TP(t_1, t_2, p)}{\partial t_2} = 0, \frac{\partial TP(t_1, t_2, p)}{\partial p} = 0 \quad (4.1.29)$$

The sufficient conditions of objective function $TP(t_1, t_2, p)$ for maxima will be verify using hessian matrix method. Consider the third order hessian matrix for $TP(t_1, t_2, p)$ at t_1, t_2, p is,

$$H(t_1, t_2, p) = \begin{pmatrix} \frac{\partial^2 TP(t_1, t_2, p)}{\partial t_1^2} & \frac{\partial^2 TP(t_1, t_2, p)}{\partial t_1 \partial t_2} & \frac{\partial^2 TP(t_1, t_2, p)}{\partial t_1 \partial p} \\ \frac{\partial^2 TP(t_1, t_2, p)}{\partial t_2 \partial t_1} & \frac{\partial^2 TP(t_1, t_2, p)}{\partial t_2^2} & \frac{\partial^2 TP(t_1, t_2, p)}{\partial t_2 \partial p} \\ \frac{\partial^2 TP(t_1, t_2, p)}{\partial p \partial t_1} & \frac{\partial^2 TP(t_1, t_2, p)}{\partial p \partial t_2} & \frac{\partial^2 TP(t_1, t_2, p)}{\partial p^2} \end{pmatrix} \quad (4.1.30)$$

$$\frac{\partial^2 TP(t_1, t_2, p)}{\partial t_1^2} < 0, \frac{\partial^2 TP(t_1, t_2, p)}{\partial t_1^2} \cdot \frac{\partial^2 TP(t_1, t_2, p)}{\partial t_2^2} - \left(\frac{\partial^2 TP(t_1, t_2, p)}{\partial t_2 \partial t_1} \right)^2 > 0, \text{ and} \quad (4.1.31)$$

$\det(H) < 0$.

Equation (4.1.28) is a nonlinear form, so we adopted the following procedure to find the optimum value of decision variables, ordering quantity of new product, buyback quantity of used product, and total profit, using mathematical software like Maple 18, Matlab, or Mathematica.

Step 1 Substitute the value of parameters in (4.1.28) except decision variables.

Step 2 Take the first order partial derivative with respect to t_1, t_2, p and equating it to zero as per (4.1.29)

Step 3 Solve (4.1.29) simultaneously and find t_1, t_2, p .

Step 4 Verify (4.1.31) at t_1, t_2, p which obtained in step-3 and find the eigen value of (4.1.30) at t_1^*, t_2^*, p^* which all are negative.

Step 5 Conditions (4.1.31) and eigen values of (4.1.30) all are not negative then go to step-1 and take the other value of parameters, repeat step 1 to step 4.

Step 6 Find total profit at t_1^*, t_2^*, p^* from (4.1.28).

Step 7 Find Q and Q_u at t_1^*, t_2^*, p^* from (4.1.7) and (4.1.14) respectively.

Step 8 Stop.

4.1.3 Numerical experiment

The proposed model is illustrated below by considering the subsequent example.

Example 4.1.1: For the numerical, graphical, and sensitivity analyses, the following numerical values for the parameters in the appropriate units are taken into consideration:

The scale demand of new product $a = 250$ units, price elasticity of new product $b = 0.4$, $\varepsilon = 0.9$, $\alpha = 100$, $\beta = 0.3$, purchasing cost $C = ₹45$ per unit, ordering cost $A = ₹100$ per order, holding cost of new product $h = ₹0.5/\text{unit/year}$, holding cost of used buyback product $h_u = ₹0.2/\text{unit/year}$, rate of depreciation of buyback product $d_r = 0.10$, $\tau = \frac{30}{365}$ year, price discount on selling price of used buyback product $p_0 = 0.5$, back order cost for new and used product $b_1 = ₹50/\text{unit}$ and $b_2 = ₹100/\text{unit}$ respectively, lost sale cost for new and used product $l_1 = ₹50/\text{unit}$ and $l_2 = 100₹50/\text{unit}$ respectively, backlogging rate is $\delta = 0.05$.

The methodology of solution described in Section 4.1.2.1 the optimal results of the proposed model are derived as Table 4.1,

Table 4.1 Optimal results of model 4.1

t_1^* (year)	t_2^* (year)	p^* (in ₹)	Q^* (units)	Q_u^* (units)	Total Profit (in ₹)
0.2417	0.2061	119.33	15.59	13.93	5471.14

Numerical validation of sufficient conditions:

From (4.1.30), hessian matrix at solution point is,

$$H(t_1^*, t_2^*, p^*) = \begin{pmatrix} -11235.95869 & 177.4249168 & -25.06601223 \\ 177.4249168 & -14499.82906 & 22.10724399 \\ -25.06601223 & 22.10724399 & -0.5423193943 \end{pmatrix}$$

$$\frac{\partial^2 TP(t_1^*, t_2^*, p^*)}{\partial t_1^2} = -11235.95869 < 0, \quad \det(H) = -7.39323 \times 10^7 < 0 \quad \text{and}$$

$$\frac{\partial^2 TP(t_1^*, t_2^*, p^*)}{\partial t_1^2} \cdot \frac{\partial^2 TP(t_1^*, t_2^*, p^*)}{\partial t_2^2} - \left(\frac{\partial^2 TP(t_1^*, t_2^*, p^*)}{\partial t_2 \partial t_1} \right)^2 = 1.6288 \times 10^7 > 0. \text{ Further, the eigen}$$

values of hessian matrix at t_1^*, t_2^*, p^* are $\lambda_1 = -14509.5$, $\lambda_2 = -11226.4$ and

$\lambda_3 = -0.453881$ all are negative, and so the objective function is maximized at the optimum value of decisions variables. (Cárdenas-Barrón and Sana[203]).

4.1.3.1 Graphical authentication of the concavity of objective functions

The concavity of profit function $TP(t_1, t_2, p)$ is shown Figure 4.1 with respect to $t_1 = 0$ to 0.5 and $p = 80$ to 170 fixed at $t_2^* = 0.261$ as below:

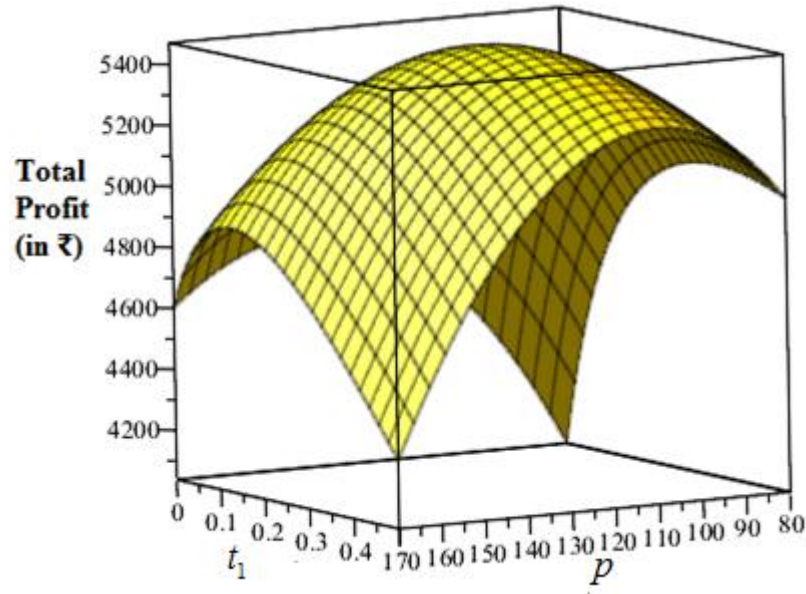


Figure 4.1 Concavity of $TP(t_1, t_2, p)$ with respect to t_1 and p for model 4.1

The concavity of profit function $TP(t_1, t_2, p)$ is shown in Figure 4.2 with respect to $t_1 = 0.1$ to 0.4 and $t_2 = 0.1$ to 0.4 fixed at $p^* = 119.33$.

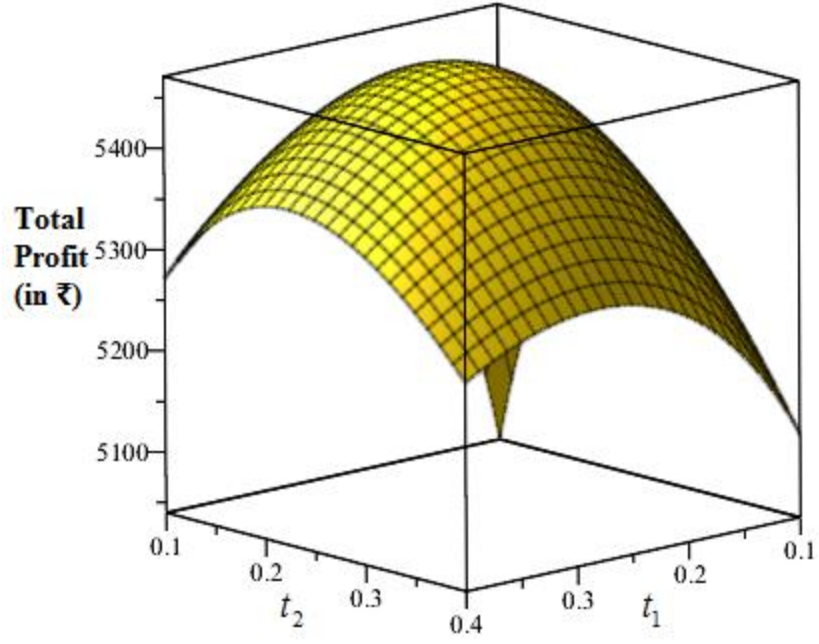


Figure 4.2 Concavity of $TP(t_1, t_2, p)$ with respect to t_2 and t_1 for model 4.1

The concavity of profit function $TP(t_1, t_2, p)$ is shown in Figure 4.3 with respect to $p = 80$ to 170 and $t_2 = 0.1$ to 0.4 fixed at $t_1^* = 0.2417$ as below:

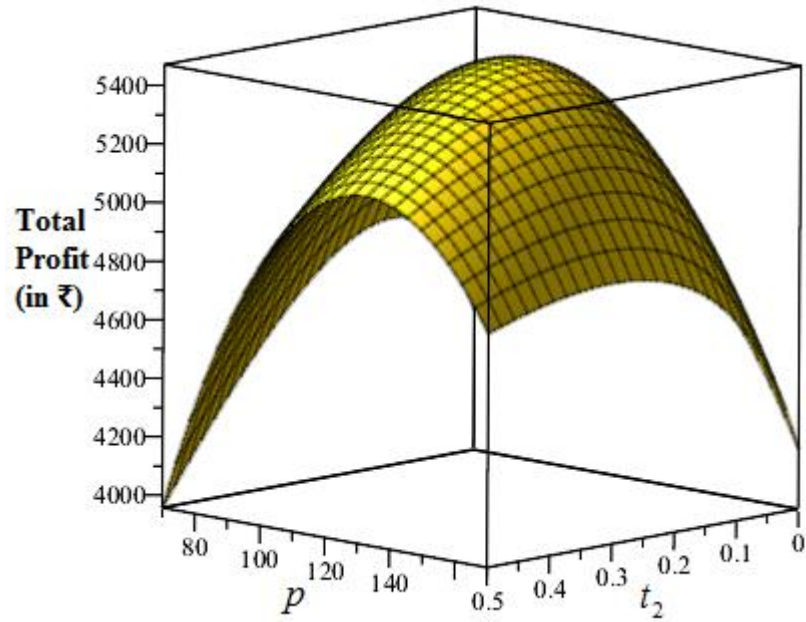


Figure 4.3 Concavity of $TP(t_1, t_2, p)$ with respect to p and t_2 for model 4.1

4.1.4 Sensitivity Analysis

Sensitivity analysis is used to alter each parameter from -20% to +20% individually while leaving the others unchanged and the optimum solutions of the proposed inventory model

are investigated. Table 4.2 gives the results of sensitivity analysis, which were derived using mathematical software like Maple 18, Matlab, or Mathematica.

Table 4.2 Sensitivity analysis of key parameters for model 4.1

Inventory Parameter	Change %	Value	p^* (in ₹)	t_1^* (year)	t_2^* (year)	Q^* (Units)	Q_u^* (Units)	Profit (in ₹)
a	-20	200	107.08	0.2820	0.1923	13.58	17.08	5006.84
	-10	225	113.26	0.2613	0.1996	14.64	15.46	5230.19
	0	250	119.33	0.2417	0.2062	15.59	13.93	5471.14
	10	275	125.30	0.2231	0.2120	16.45	12.50	5729.02
	20	300	131.16	0.2056	0.2171	17.22	11.15	6003.15
b	-20	0.32	154.11	0.1477	0.2322	18.41	6.60	6897.26
	-10	0.36	134.48	0.1979	0.2209	17.17	10.50	6048.93
	0	0.4	119.33	0.2417	0.2062	15.59	13.93	5471.14
	10	0.44	107.46	0.2791	0.1922	13.99	16.89	5070.09
	20	0.48	98.05	0.3103	0.1800	12.45	19.42	4786.90
α	-20	80	105.25	0.2322	0.1987	15.80	10.42	3886.95
	-10	90	112.13	0.2373	0.2019	15.69	11.41	4664.72
	0	100	119.33	0.2417	0.2062	15.59	13.93	5471.14
	10	110	126.81	0.2457	0.2112	15.51	16.52	6307.34
	20	120	134.53	0.2493	0.2166	15.44	19.17	7174.34
β	-20	0.24	118.08	0.2704	0.1975	16.18	14.96	5512.89
	-10	0.27	118.72	0.2553	0.2021	15.87	14.41	5490.99
	0	0.3	119.33	0.2417	0.2062	15.59	13.93	5471.14
	10	0.33	119.92	0.2295	0.2100	15.33	13.50	5453.11
	20	0.36	120.47	0.2183	0.2135	15.10	13.12	5436.69
A	-20	80	119.50	0.2322	0.1993	15.50	13.31	5516.63
	-10	90	119.41	0.2370	0.2028	15.32	13.63	5493.68
	0	100	119.33	0.2417	0.2062	15.59	13.93	5471.14
	10	110	119.26	0.2463	0.2095	15.85	14.23	5449.02
	20	120	119.18	0.2509	0.2128	16.11	14.53	5427.27
h	-20	0.4	119.32	0.2418	0.2062	15.60	13.94	5471.35
	-10	0.45	119.33	0.2418	0.2062	15.60	13.93	5471.25
	0	0.5	119.33	0.2417	0.2062	15.59	13.93	5471.14
	10	0.55	119.34	0.2416	0.2062	15.59	13.93	5471.04
	20	0.6	119.34	0.2416	0.2062	15.59	13.93	5470.94
d	-20	0.08	118.42	0.2426	0.2051	15.63	14.09	5505.63
	-10	0.09	118.88	0.2422	0.2057	15.61	14.00	5488.34
	0	0.1	119.33	0.2417	0.2062	15.59	13.93	5471.14
	10	0.11	119.79	0.2412	0.2067	15.57	13.86	5454.05
	20	0.12	120.25	0.2408	0.2073	15.56	13.78	5437.05
τ	-20	0.065753	118.60	0.2281	0.1911	14.67	13.63	5557.41
	-10	0.073973	118.96	0.2351	0.1988	15.14	13.80	5513.30
	0	0.082192	119.33	0.2417	0.2062	15.59	13.93	5471.14
	10	0.090411	119.71	0.2479	0.2134	16.00	14.04	5430.79

Inventory Parameter	Change %	Value	p^* (in ₹)	t_1^* (year)	t_2^* (year)	Q^* (Units)	Q_u^* (Units)	Profit (in ₹)
	20	0.09863	120.10	0.2536	0.2204	16.42	14.13	5392.07
p_0	-20	0.4	95.15	0.2724	0.1908	17.44	15.30	4923.47
	-10	0.45	105.96	0.2575	0.1980	16.53	14.64	5178.14
	0	0.5	119.33	0.2417	0.2062	15.59	13.93	5471.14
	10	0.55	136.24	0.2247	0.2154	14.62	13.15	5813.88
	20	0.6	158.21	0.2062	0.2259	13.62	12.26	6223.03
ε	-20	0.72	119.02	0.2569	0.2014	16.07	14.30	5500.70
	-10	0.81	119.17	0.2490	0.2038	15.82	14.11	5485.48
	0	0.9	119.33	0.2417	0.2062	15.59	13.93	5471.14
	10	0.99	119.50	0.2349	0.2084	15.38	13.76	5457.59
	20	1.08	119.66	0.2284	0.2106	15.19	13.61	5444.75
b_1	-20	40	119.84	0.2371	0.2163	15.80	14.09	5489.27
	-10	45	119.58	0.2395	0.2111	15.70	14.00	5480.03
	0	50	119.33	0.2417	0.2062	15.59	13.93	5471.14
	10	55	119.09	0.2439	0.2015	15.50	13.86	5462.61
	20	60	118.87	0.2459	0.1970	15.41	13.80	5454.41
l_2	-20	80	119.35	0.2413	0.2072	15.62	13.96	5473.06
	-10	90	119.34	0.2415	0.2067	15.60	13.94	5472.10
	0	100	119.33	0.2417	0.2062	15.59	13.93	5471.14
	10	110	119.33	0.2419	0.2057	15.58	13.92	5470.19
	20	120	119.32	0.2421	0.2052	15.57	13.91	5469.25
δ	-20	0.04	119.48	0.2402	0.2092	15.66	13.99	5476.98
	-10	0.045	119.40	0.2410	0.2077	15.63	13.96	5474.04
	0	0.05	119.33	0.2417	0.2062	15.59	13.93	5471.14
	10	0.055	119.26	0.2424	0.2047	15.56	13.91	5468.28
	20	0.06	119.19	0.2431	0.2032	15.53	13.88	5465.46

As per the tabular values, the key observations made as below:

- Optimal selling price highly increases when system parameters a, α and p_0 increases. If increases in β, d, τ, h and ε then selling price slightly increase. The parameters b, C, A, b_1, l_1 and δ are negative proportional with respect to selling price, in which increases in b result to selling price extremely decreases.
- If we increase in a up to 40% then total profit increases around 20%. The parameter α is highly sensitive to total profit, if increases in α up to 40% then total profit increase up to 85%. The higher discount rate also positive to profit, profit will be increases 25% to 30%, if p_0 increases -20% to 20%, however, the parameters $b, \beta, d, A, \varepsilon$ increases -20% to 20%, total profit will decrease up to 5%. Total profit slightly decreases for the increases, if decreases in h, b_1, l_2, δ .

- The positive inventory period will be noticeably increase if increases in b and τ . If increase in α, A, b_1, δ and l_2 then positive cycle time increases moderately. The parameters a, β, h, h_u, d_r and ε increases then positive cycle time decreases. If increases in p_0 then positive inventory cycle thoroughly decreases.
- When the values of parameters $a, \alpha, \beta, A, d, \varepsilon$ and τ increases, the optimal shortages period will also increase slightly, but shortages period extremely increase for increases in p_0 , on the other side increases parameter $b_1, b_2, l_1, l_2, \delta$ shortages period slightly decreases.
- Order quantity of new product increases supremely if increases in a and τ . The parameters β and A increases result to ordering quantity of new product increases moderately, increases in b and p_0 then ordering quantity of new product products noticeably decreases. The buyback quantity of used product is extremely decreases for increases in a and p_0 , highly increases for b and α .

4.2 Optimal inventory strategy for deteriorating products for which shortages are partially backlogged

In the management of inventory systems, keeping and restoring inventories of deteriorating products has become an important challenge. This proposed model deals with deteriorating products in which the retailer fulfills the demand by selling new as well as used buyback products, but unsatisfied demand is partially backlogged.

4.2.1 Notations and Assumptions

4.2.1.1 Notations

The notations of proposed model 4.2 are same as Section 4.1.1.1 with below additionally notations.

θ Constant rate of deterioration for new product.

θ_u Constant rate of deterioration for used buyback product.

4.2.1.2 Assumptions

The assumptions of proposed model 4.2 are similar to those in Section 4.1.1.2, with the following additional assumptions:

1. The product is deteriorating by nature, with a constant rate of deterioration for both new and used buyback products. Rate of deterioration is θ for new product and θ_u is the rate of deterioration for used buyback product with $\theta_u \geq \theta$.
2. Replacement or repair of deteriorating product during the period is not allowed.

4.2.2 Mathematical Formulation

This section involved a mathematical modelling of inventory system for the deterioration new products and used products in which shortages are partially backlogged. First we derive the retailer's sales revenue and associated costs for new products. Initially, retailer received Q quantity of new products; stock level declines and sinking to zero by integrated effects of demand and constant rate of deterioration. As per Ghare and Schrader[5], the position of inventory of the new products at time t over the period $[0, t_1]$ is expressed in the form of differential equation as (4.2.1).

$$\frac{dI_1(t)}{dt} + \theta I_1(t) = -R_n(p, t), 0 \leq t \leq t_1 \quad (4.2.1)$$

Using the boundary condition $I_1(t_1) = 0$, the solution of (4.2.1) is,

$$I_1(t) = \frac{e^{-\theta t} a p^{-b}}{\varepsilon - \theta} \left(e^{-(\varepsilon - \theta)t} - e^{-(\varepsilon - \theta)t_1} \right) \quad (4.2.2)$$

At time t_1 , the inventory level became zero and shortages had arisen. In the instance of a stock out, some customers may be willing to wait for a delayed shipment, while others who have an urgent need may depart to look for a new provider. During the shortages span, the required stock of inventory is solely based on demand, and a fraction $e^{-\delta(t_2 - t)}$ of the demand is backlogged, where $(t_2 - t)$ is waiting period up to next delivery. The status of inventory of new products during stock-out period $[0, t_2]$ can be expressed by the differential equation,

$$\frac{dI_2(t)}{dt} = -R_n(p, t)e^{-\delta(t_2-t)}, 0 \leq t \leq t_2, \text{ with } I_2(0) = 0 \quad (4.2.3)$$

The solution of (4.2.3) is,

$$I_2(t) = \frac{ap^{-b}e^{-\delta t_2}}{\delta} (1 - e^{\delta t}), 0 \leq t \leq t_2 \quad (4.2.4)$$

The highest available inventory for new products is,

$$IM = I_1(0) = \frac{ap^{-b}}{\varepsilon - \theta} (1 - e^{-(\varepsilon - \theta)t_1}) \quad (4.2.5)$$

The highest backordered units of new products are,

$$IB = -I_2(t_2) = \frac{ap^{-b}}{\delta} (1 - e^{-\delta t_2}) \quad (4.2.6)$$

Thus, the ordering quantity of new products over the cycle time can be deduced as,

$$Q = IM + IB = \frac{ap^{-b}}{\varepsilon - \theta} (1 - e^{-(\varepsilon - \theta)t_1}) + \frac{ap^{-b}}{\delta} (1 - e^{-\delta t_2}) \quad (4.2.7)$$

The inventory level of used products, which depends upon the buyback rate. As per the assumptions retailer start to collect the used product at time τ , and same time start to sells it, and the inventory level of used products reduces due to united effects of demand and deterioration, hence inventory level $I_{u1}(t)$ of used products in the period $[\tau, t_1]$ can be introduced through the differential equation,

$$\frac{dI_{u1}(t)}{dt} + \theta_u I_{u1}(t) = -R_u(p, t), \tau \leq t \leq t_1 \quad (4.2.8)$$

At the boundary condition $I_{u1}(t_1) = 0$, the solution of (4.2.8) is obtained as,

$$I_{u1}(t) = \frac{\alpha\beta t}{\theta_u} - \frac{pp_0}{\theta_u} - \frac{\alpha\beta}{\theta_u^2} - \frac{\alpha}{\theta_u} + \frac{p}{\theta_u} - e^{-\theta_u(t-t_1)} \left(\frac{\alpha\beta t_1}{\theta_u} - \frac{pp_0}{\theta_u} - \frac{\alpha\beta}{\theta_u^2} - \frac{\alpha}{\theta_u} + \frac{p}{\theta_u} \right) \quad (4.2.9)$$

The required stock during the shortages period which only depends on current demand as well as portion $e^{-\delta(t_2-t)}$ of the demand is backlogged, where $(t_2 - t)$ is waiting period up to next delivery. It was proposed that since customers dislike waiting, the percentage of consumers who decide to take back orders be a decreasing form of waiting time (Abad[33]). The inventory level for used buyback product during the stock out period is demonstrated by the differential equation,

$$\frac{dI_{u2}(t)}{dt} = -R_u(p, t)e^{-\delta(t_2-t)}, 0 \leq t \leq t_2 \text{ with } I_{u2}(0) = 0 \quad (4.2.10)$$

The solution of the differential equation (4.2.10) is given by,

$$I_{u2}(t) = \frac{\alpha - p(1-p_0)}{\delta} (e^{-\delta t_2} - e^{\delta(t-t_2)}), 0 \leq t \leq t_2 \quad (4.2.11)$$

The highest positive inventory level for used products at $t = \tau$ is,

$$IM_u = I_{u1}(\tau) = \frac{\alpha\beta\tau}{\theta_u} - \frac{pp_0}{\theta_u} - \frac{\alpha\beta}{\theta_u^2} - \frac{\alpha}{\theta_u} + \frac{p}{\theta_u} - e^{-\theta_u(\tau-t_1)} \left(\frac{\alpha\beta t_1}{\theta_u} - \frac{pp_0}{\theta_u} - \frac{\alpha\beta}{\theta_u^2} - \frac{\alpha}{\theta_u} + \frac{p}{\theta_u} \right) \quad (4.2.12)$$

The highest units of backordered of used product are,

$$IB_u = -I_{u2}(t_2) = \frac{\alpha - p(1-p_0)}{\delta} (1 - e^{-\delta t_2}) \quad (4.2.13)$$

Thus, the total buyback quantity of used product from (4.2.12) and (4.2.13) is given by,

$$Q_u = \left[e^{-\theta_u(\tau-t_1)} \left(\frac{\alpha\beta t_1}{\theta_u} - \frac{pp_0}{\theta_u} - \frac{\alpha\beta}{\theta_u^2} - \frac{\alpha}{\theta_u} + \frac{p}{\theta_u} \right) - \left(\frac{\alpha\beta\tau}{\theta_u} - \frac{pp_0}{\theta_u} - \frac{\alpha\beta}{\theta_u^2} - \frac{\alpha}{\theta_u} + \frac{p}{\theta_u} \right) \right] - \left[-\frac{\alpha - p(1-p_0)}{\delta} (1 - e^{-\delta t_2}) \right] \quad (4.2.14)$$

To analyse the total profit of retailer from new product, we determine the following various components:

Retailer's sales revenue generated from new products:

$$SR_n = \frac{p}{t_1+t_2} \left(\int_0^{t_1} ap^{-b} e^{-\varepsilon t} dt + \int_0^{t_2} ap^{-b} e^{-\delta(t_2-t)} dt \right) \quad (4.2.15)$$

$$\text{Cost of purchasing of new products: } PC_n = \frac{CQ}{t_1+t_2} \quad (4.2.16)$$

$$\text{Holding cost for new products: } HC_n = \frac{h_1}{t_1+t_2} \int_0^{t_1} I_1(t) dt \quad (4.2.17)$$

$$\text{New products ordering cost: } OC_n = \frac{A}{t_1+t_2} \quad (4.2.18)$$

Shortage cost due to backordered for new products: $BC_n = \frac{b_1}{t_1 + t_2} \int_0^{t_2} -I_2(t)dt$ (4.2.19)

Lost sales cost for new products: $LS_n = \frac{l_1}{t_1 + t_2} \int_0^{t_2} ap^{-b}(1 - e^{-\delta(t_2-t)})dt$ (4.2.20)

Total profit for new product during the cycle time is,

$$TP_n(t_1, t_2, p) = SR_n - OC_n - HC_n - PC_n - BC_n - LS_n \quad (4.2.21)$$

Now, to evaluate total profit from the selling of buyback used product, we calculate all the components are listed below:

Sales revenue from used buyback products:

$$SR_u = \frac{p(1-p_0)}{t_1 + t_2} \left(\int_{\tau}^{t_1} (\alpha(1-\beta t) - p(1-p_0))dt + \int_0^{t_2} (\alpha - p(1-p_0)e^{-\delta(t_2-t)})dt \right) \quad (4.2.22)$$

Purchasing cost for used products: $PC_u = \frac{C(1-d_r)Q_u}{(t_1 + t_2) - \tau}$ (4.2.23)

Cost of holding of used buyback products: $HC_u = \frac{h_2}{t_1 + t_2} \int_{\tau}^{t_1} I_{u1}(t)dt$ (4.2.24)

Cost due to backordered of used products:

$$BC_u = \frac{b_2}{t_1 + t_2} \int_0^{t_2} -I_{u2}(t)dt \quad (4.2.25)$$

Cost occurs due to lost sale of used products:

$$LS_u = \frac{l_2}{t_1 + t_2} \int_0^{t_2} (\alpha - p(1-p_0))(1 - e^{-\delta(t_2-t)})dt \quad (4.2.26)$$

Total profit of retailer earn from used buyback product during the cycle time is,

$$TP_u(t_1, t_2, p) = SR_u - PC_u - HC_u - BC_u - LS_u \quad (4.2.27)$$

Therefore, the total profit from the both product is given by from (4.2.21) and (4.2.27)

$$TP(t_1, t_2, p) = (SR_n - OC_n - PC_n - HC_n - BC_n - LS_n) + (SR_u - PC_u - HC_u - BC_u - LS_u) \quad (4.2.28)$$

$$TP(t_1, t_2, p) = \left\{ \left[\frac{p}{t_1 + t_2} \left(\int_0^{t_1} ap^{-b} e^{-\varepsilon t} dt + \int_0^{t_2} ap^{-b} e^{-\delta(t_2-t)} dt \right) - \frac{A}{t_1 + t_2} - \frac{CQ}{t_1 + t_2} - \frac{h}{t_1 + t_2} \int_0^{t_1} I_1(t) dt \right. \right. \\ \left. \left. - \frac{b_1}{t_1 + t_2} \int_0^{t_2} -I_2(t) dt - \frac{l_1}{t_1 + t_2} \int_0^{t_2} ap^{-b} (1 - e^{-\delta(t_2-t)}) dt \right] \right. \\ \left. + \left[\frac{p(1-p_0)}{t_1 + t_2} \left(\int_{\tau}^{t_1} (\alpha(1-\beta t) - p(1-p_0)) dt + \int_0^{t_2} (\alpha - p(1-p_0) e^{-\delta(t_2-t)}) dt \right) \right] \right. \\ \left. + \left[-\frac{C(1-d_r)Q_u}{t_1 + t_2 - \tau} - \frac{h_u}{t_1 + t_2} \int_{\tau}^{t_1} I_{u1}(t) dt \right. \right. \\ \left. \left. - \frac{b_2}{t_1 + t_2} \int_0^{t_2} -I_{u2}(t) dt - \frac{l_2}{t_1 + t_2} \int_0^{t_2} (\alpha - p(1-p_0)(1 - e^{-\delta(t_2-t)}) dt \right] \right\}$$

In above expression, to evaluate the optimal value of decision variables t_1, t_2 and p which maximize $TP(t_1, t_2, p)$.

4.2.2.1 Solution technique to determine the optimal solution

The analytical method for single objective problem to be apply for finding the optimal solution of positive cycle time, shortages period and selling price such that objective function is maximize.

The necessary conditions for maximize the total profit function given by (4.2.28) are

$$\frac{\partial TP(t_1, t_2, p)}{\partial t_1} = 0, \frac{\partial TP(t_1, t_2, p)}{\partial t_2} = 0, \frac{\partial TP(t_1, t_2, p)}{\partial p} = 0 \quad (4.2.29)$$

To verify the sufficient conditions for maximize $TP(t_1, t_2, p)$ by using the hessian matrix method, consider hessian matrix,

$$H(t_1, t_2, p) = \begin{pmatrix} \frac{\partial^2 TP(t_1, t_2, p)}{\partial t_1^2} & \frac{\partial^2 TP(t_1, t_2, p)}{\partial t_1 \partial t_2} & \frac{\partial^2 TP(t_1, t_2, p)}{\partial t_1 \partial p} \\ \frac{\partial^2 TP(t_1, t_2, p)}{\partial t_2 \partial t_1} & \frac{\partial^2 TP(t_1, t_2, p)}{\partial t_2^2} & \frac{\partial^2 TP(t_1, t_2, p)}{\partial t_2 \partial p} \\ \frac{\partial^2 TP(t_1, t_2, p)}{\partial p \partial t_1} & \frac{\partial^2 TP(t_1, t_2, p)}{\partial p \partial t_2} & \frac{\partial^2 TP(t_1, t_2, p)}{\partial p^2} \end{pmatrix} \quad (4.2.30)$$

The sufficient conditions of objective function is maximize are, (4.2.31)

$$\frac{\partial^2 TP(t_1, t_2, p)}{\partial t_1^2} < 0, \frac{\partial^2 TP(t_1, t_2, p)}{\partial t_1^2} \cdot \frac{\partial^2 TP(t_1, t_2, p)}{\partial t_2^2} - \left(\frac{\partial^2 TP(t_1, t_2, p)}{\partial t_2 \partial t_1} \right)^2 > 0, \text{ and}$$

$$\det(H) < 0.$$

Equation (4.2.28) is a nonlinear form and it may be difficult to solve directly, so we accept the below given methodology to calculate the optimum value of decision variables, ordering quantity of new product, buyback quantity of used product, and total profit, using mathematical software like Maple 18, Matlab, or Mathematica.

Step 1 Input the all parametric values in (4.2.28) except decision variables.

Step 2 Obtains the first order partial derivative with respect to t_1, t_2, p and equating it to zero as per (4.2.29).

Step 3 Solve (4.1.29) simultaneously and find t_1, t_2, p .

Step 4 Check the conditions of (4.2.31) at t_1, t_2, p which obtained in step-3 and find the eigen value of (4.2.30) at t_1^*, t_2^*, p^* which all are negative. If it is satisfied, values obtained in step 3 are optimum value of decision variables.

Step 5 If not satisfied (4.2.31) and all eigen values of (4.2.30) are not negative then go to step-1 and take the different values of parameters and repeat step 1 to step 4.

Step 6 Find total profit at t_1^*, t_2^*, p^* from (4.2.28).

Step 7 Find Q and Q_u at t_1^*, t_2^*, p^* from (4.2.7) and (4.2.14) respectively.

Step 8 Stop.

4.2.3 Numerical experiment

The proposed model is illustrated below by considering the following example. The model uses the numerical data from the literature with proper units.

Example 4.2.1: The following numerical values of the parameter in proper unit were considered as input for numerical, graphical and sensitivity analysis of the model.

The scale demand of new product $a = 250$ units, price elasticity of new product $b = 0.4$, $\alpha = 100$, $\beta = 0.3$, purchasing cost $C = ₹45$ per unit, ordering cost $A = ₹100$ per order, holding cost of new product $h = ₹0.5/\text{unit}/\text{year}$, holding cost of used buyback product $h_u = ₹0.2/\text{unit}/\text{year}$, rate of depreciation of buyback product $d_r = 0.10$, $\tau = \frac{30}{365}$ year, price

discount on selling price of used buyback product $p_0 = 0.5$, $\varepsilon = 0.9$, back order cost for new and used product $b_1 = ₹50/\text{unit}$ and $b_2 = ₹100/\text{unit}$ respectively, lost sale cost for new and used product $l_1 = ₹50/\text{unit}$ and $l_2 = ₹100/\text{unit}$ respectively, backlogging rate is $\delta = 0.05$, $\theta = 0.01$ and $\theta_u = 0.02$. As per the solution process step's presented in Section 4.2.2.2, the optimal results of the proposed model are deduced as,

Table 4.3 Optimal results of model 4.2

t_1^* (year)	t_2^* (year)	p^* (in ₹)	Q^* (units)	Q_u^* (units)	Total Profit (in ₹)
0.2422	0.2061	119.29	14.61	13.96	5471.20

Note that total cycle time $T^* = t_1^* + t_2^* = 0.45$ year.

Numerical validation of sufficient conditions:

From (4.2.30) hessian matrix at solution point is

$$H(t_1^*, t_2^*, p^*) = \begin{pmatrix} -11191.61 & 173.86 & -25.16 \\ 173.86 & -14492.33 & 22.15 \\ -25.16 & 22.15 & -0.54 \end{pmatrix}$$

$$\frac{\partial^2 TP(t_1^*, t_2^*, p^*)}{\partial t_1^2} = 11191.61 < 0, \det(H) = -7.34979 \times 10^7 < 0 \text{ and}$$

$$\frac{\partial^2 TP(t_1^*, t_2^*, p^*)}{\partial t_1^2} \cdot \frac{\partial^2 TP(t_1^*, t_2^*, p^*)}{\partial t_2^2} - \left(\frac{\partial^2 TP(t_1^*, t_2^*, p^*)}{\partial t_2 \partial t_1} \right)^2 = 1.62162 \times 10^8 > 0. \text{ Further, the eigen}$$

values of hessian matrix at t_1^*, t_2^*, p^* are $\lambda_1 = -13298 < 0$, $\lambda_2 = -11089 < 0$ and $\lambda_3 = -0.45 < 0$ all are negative, and so the objective function is maximized at the optimum value of decisions variables. (Cardenas-Barron and Sana[203]).

4.2.3.1 Graphical authentication of the concavity of objective functions

The concavity behaviour of objective function as two-dimensional graphs is shown in

Figure 4.4, Figure 4.5 and Figure 4.6.

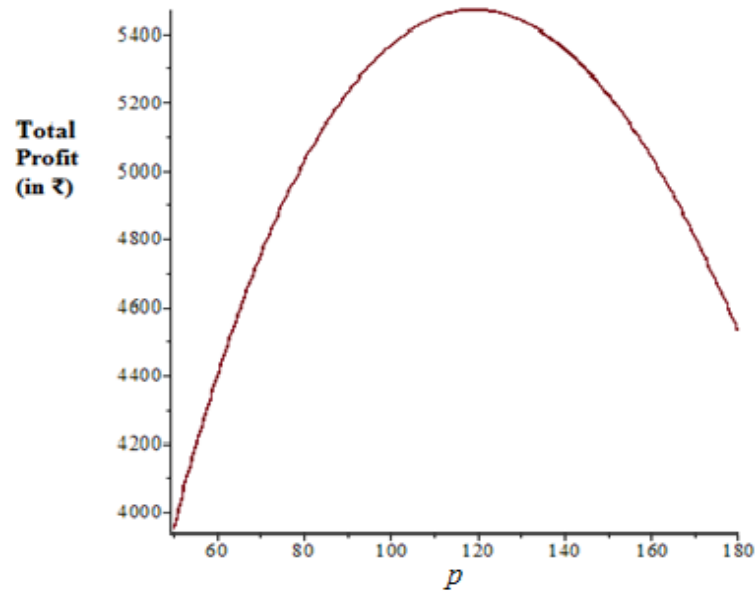


Figure 4.4 Total profit vs Selling price

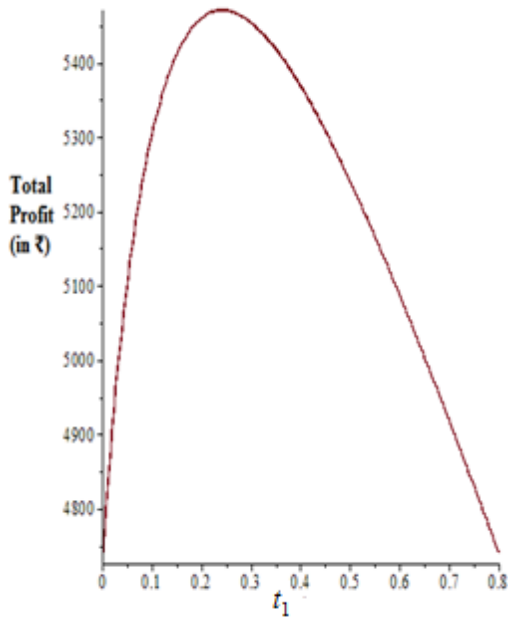


Figure 4.5 Total profit vs Positive cycle time

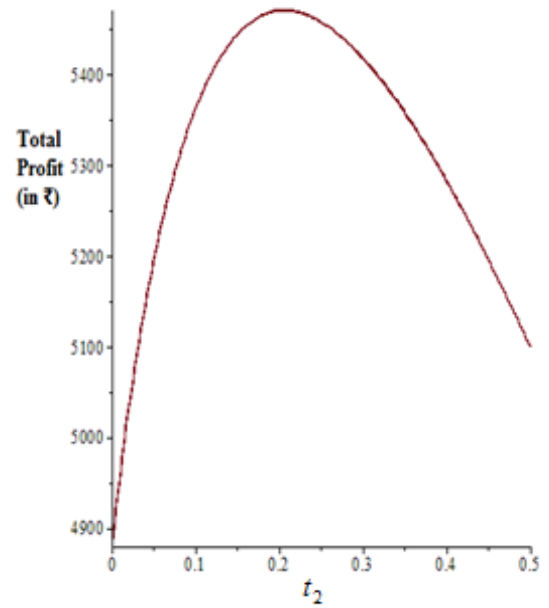


Figure 4.6 Total Profit vs Shortages period

The concavity behaviour of objective function as three-dimensional graphs is shown in Figure 4.7, Figure 4.8 and Figure 4.9.

The concavity of $TP(t_1, t_2, p)$ is shown in Figure 4.7 with respect to $t_1 = 0.1$ to 0.4 and $t_2 = 0.1$ to 0.4 fixed at $p^* = 119.28$ as below:

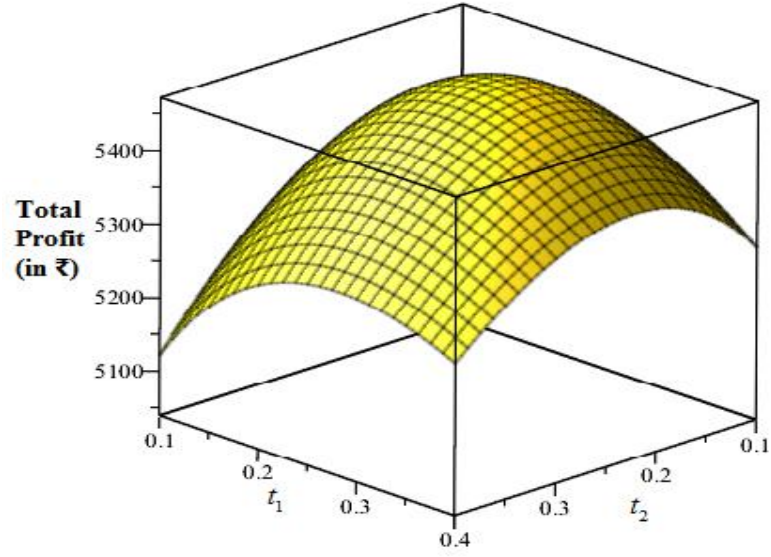


Figure 4.7 Concavity of $TP(t_1, t_2, p)$ with respect to t_1 and t_2 for model 4.2

The concavity $TP(t_1, t_2, p)$ is shown in Figure 4.8 with respect to $p = 70$ to 170 and $t_2 = 0$ to 0.5 fixed at $t_1 = 0.2421572$ as below:

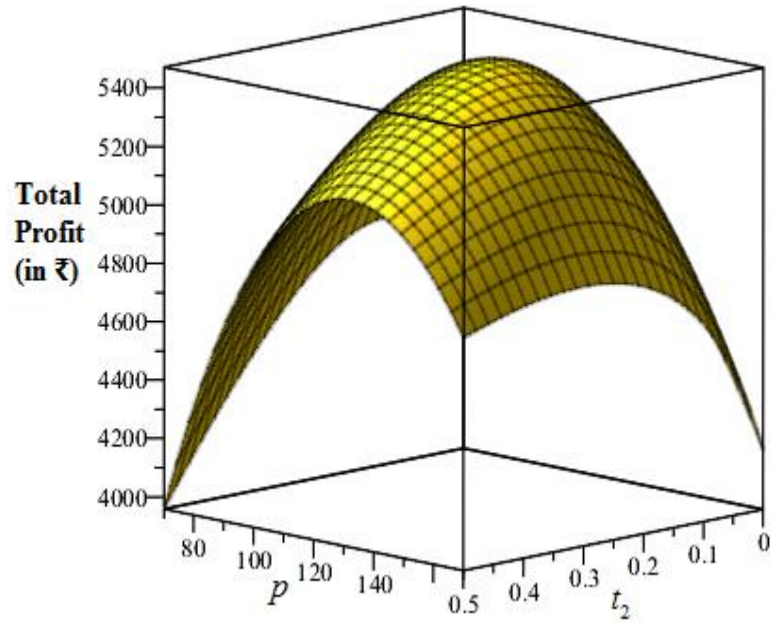


Figure 4.8 Concavity of $TP(t_1, t_2, p)$ with respect to p and t_2 for model 4.2

The concavity of $TP(t_1, t_2, p)$ is also shown in Figure 4.9 with respect to $t_1 = 0$ to 0.5 and $p = 80$ to 170 fixed at $t_2 = 0.21$ as below:

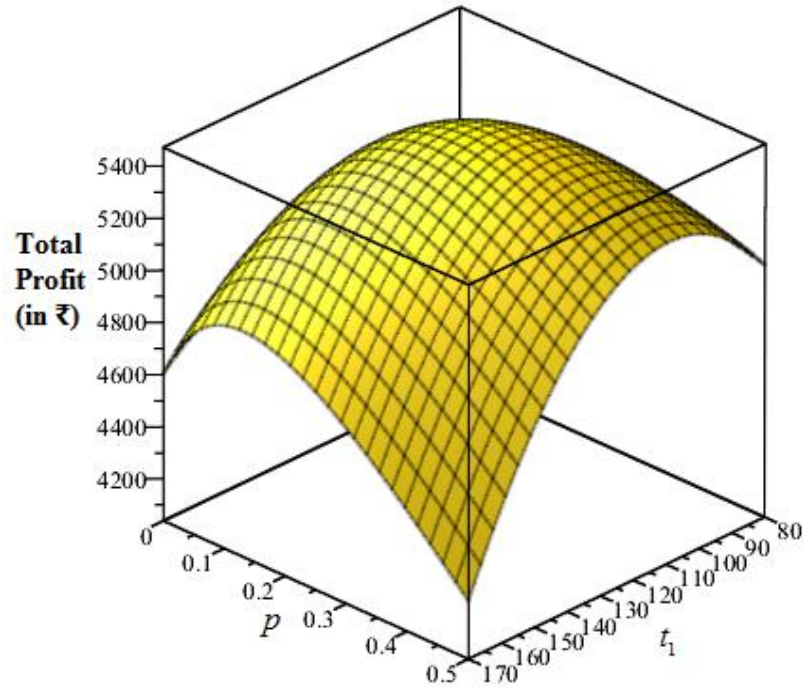


Figure 4.9 Concavity of $TP(t_1, t_2, p)$ with respect to t_1 and p for model 4.2

4.2.4 Sensitivity Analysis

Table 4.4 shows the effects in optimal results due to the sensitivity of system parameters between -20% and +20% using the mathematical software like Maple 18 or Matlab or Mathematica. One parameter is taken at a time while the values of the other parameters are left fixed when computing the sensitivity of a parameter in example 4.2.1.

Table 4.4 Sensitivity analysis of key parameters for model 4.2

Inventory Parameter	Change %	Value	p^* (in ₹)	t_1^* (year)	t_2^* (year)	Q^* (Units)	Q_u^* (Units)	Profit (in ₹)
a	-20	200	107.00	0.2830	0.1920	13.61	17.14	5007.52
	-10	225	113.20	0.2620	0.1994	14.66	15.50	5230.50
	0	250	119.29	0.2422	0.2061	15.61	13.96	5471.20
	10	275	125.27	0.2234	0.2120	16.47	12.52	5728.85
	20	300	131.14	0.2057	0.2171	17.23	11.16	6002.83
b	-20	0.32	154.13	0.1475	0.2323	18.41	6.57	6896.79
	-10	0.36	134.47	0.1979	0.2209	17.18	10.51	6048.57
	0	0.4	119.29	0.2422	0.2061	15.61	13.96	5471.20
	10	0.44	107.39	0.2800	0.1919	14.02	16.95	5070.70
	20	0.48	97.95	0.3118	0.1796	12.49	19.51	4788.16
α	-20	80	105.23	0.2323	0.1987	15.82	8.96	3886.60
	-10	90	112.10	0.2376	0.2019	15.71	11.43	4664.55
	0	100	119.29	0.2422	0.2061	15.61	13.96	5471.20

Inventory Parameter	Change %	Value	p^* (in ₹)	t_1^* (year)	t_2^* (year)	Q^* (Units)	Q_u^* (Units)	Profit (in ₹)
	10	110	126.75	0.2463	0.2110	15.53	16.56	6307.57
	20	120	134.47	0.2499	0.2165	15.46	19.22	7174.80
β	-20	0.24	118.02	0.2711	0.1974	16.21	15.00	5513.12
	-10	0.27	118.67	0.2558	0.2019	15.90	14.45	5491.12
	0	0.3	119.29	0.2422	0.2061	15.61	13.96	5471.20
	10	0.33	119.88	0.2298	0.2099	15.35	13.53	5453.09
	20	0.36	120.44	0.2186	0.2134	15.12	13.14	5436.63
C	-20	36	126.08	0.2279	0.2158	15.20	12.65	5485.80
	-10	40.5	122.62	0.2351	0.2109	15.41	13.31	5475.93
	0	45	119.29	0.2422	0.2061	15.61	13.96	5471.20
	10	49.5	116.08	0.2490	0.2014	15.82	14.59	5465.28
	20	54	113.00	0.2557	0.1969	16.02	15.21	5458.98
A	-20	80	119.46	0.2326	0.1992	15.07	13.34	5516.62
	-10	90	119.37	0.2374	0.2027	15.34	13.65	5493.70
	0	100	119.29	0.2422	0.2061	15.61	13.96	5471.20
	10	110	119.21	0.2468	0.2094	15.88	14.26	5449.07
	20	120	119.14	0.2514	0.2127	16.14	14.56	5427.34
h	-20	0.4	119.28	0.2423	0.2060	15.62	13.96	5471.39
	-10	0.45	119.28	0.2422	0.2061	15.62	13.96	5471.28
	0	0.5	119.29	0.2422	0.2061	15.61	13.96	5471.20
	10	0.55	119.29	0.2421	0.2061	15.61	13.96	5471.08
	20	0.6	119.30	0.2420	0.2061	15.61	13.96	5470.99
h_u	-20	0.16	119.28	0.2422	0.2061	15.61	13.96	5471.23
	-10	0.18	119.28	0.2422	0.2061	15.61	13.96	5471.19
	0	0.2	119.29	0.2422	0.2061	15.61	13.96	5471.20
	10	0.22	119.28	0.2421	0.2061	15.61	13.96	5471.17
	20	0.24	119.29	0.2421	0.2061	15.61	13.96	5471.14
d	-20	0.08	118.37	0.2431	0.2050	15.65	14.12	5505.70
	-10	0.09	118.83	0.2426	0.2055	15.63	14.04	5488.39
	0	0.10	119.29	0.2422	0.2061	15.61	13.96	5471.20
	10	0.11	119.75	0.2417	0.2066	15.60	13.88	5454.07
	20	0.12	120.21	0.2412	0.2072	15.58	13.80	5437.06
τ	-20	0.06575	118.55	0.2285	0.1910	14.69	13.66	5557.58
	-10	0.07397	118.92	0.2356	0.1987	15.16	13.82	5513.40
	0	0.08219	119.29	0.2422	0.2061	15.61	13.96	5471.20
	10	0.09041	119.67	0.2483	0.2133	16.04	14.07	5430.74
	20	0.09863	120.06	0.2540	0.2203	16.44	14.15	5391.98
p_0	-20	0.4	95.10	0.2730	0.1906	17.47	15.34	4923.64
	-10	0.45	105.91	0.2581	0.1979	16.56	14.68	5178.24
	0	0.5	119.29	0.2422	0.2061	15.61	13.96	5471.20
	10	0.55	136.20	0.2250	0.2153	14.64	13.17	5813.86
	20	0.6	158.17	0.2142	0.2321	14.20	12.52	6222.97
ε	-20	0.72	118.97	0.2574	0.2012	16.10	14.33	5500.77
	-10	0.81	119.13	0.2495	0.2037	15.84	14.14	5485.54
	0	0.9	119.29	0.2422	0.2061	15.61	13.96	5471.20

Inventory Parameter	Change %	Value	p^* (in ₹)	t_1^* (year)	t_2^* (year)	Q^* (Units)	Q_u^* (Units)	Profit (in ₹)
	10	0.99	119.45	0.2353	0.2084	15.40	13.79	5457.63
	20	1.08	119.62	0.2288	0.2105	15.21	13.63	5444.75
θ	-20	0.008	119.28	0.2423	0.2061	15.61	13.96	5471.37
	-10	0.009	119.28	0.2422	0.2061	15.61	13.96	5471.28
	0	0.01	119.29	0.2422	0.2061	15.61	13.96	5471.20
	10	0.011	119.29	0.2421	0.2061	15.61	13.96	5471.10
	20	0.012	119.30	0.2420	0.2061	15.61	13.96	5470.99
θ_u	-20	0.016	119.30	0.2420	0.2062	15.61	13.95	5470.98
	-10	0.018	119.30	0.2421	0.2061	15.61	13.95	5471.07
	0	0.02	119.29	0.2422	0.2061	15.61	13.96	5471.20
	10	0.022	119.28	0.2423	0.2061	15.62	13.97	5471.28
	20	0.024	119.27	0.2424	0.2060	15.62	13.97	5471.39
δ	-20	0.04	119.43	0.2407	0.2091	15.68	14.02	5477.02
	-10	0.045	119.36	0.2414	0.2076	15.65	13.99	5474.09
	0	0.05	119.29	0.2422	0.2061	15.61	13.96	5471.20
	10	0.055	119.22	0.2429	0.2046	15.58	13.93	5468.31
	20	0.06	119.15	0.2436	0.2031	15.55	13.91	5465.51

The following observations made from the sensitivity of parameters evaluated in Table and figure.

- The positive impact on profit of retailer for higher value of scale demand of new products a , if a increases -20% to 20% then profit also increases up to 25%. The parameter α is highly sensitive to retailer's profit, if α increases -20% to 20% then profit extremely increases up to 85% and -20% to 20% increases in p_0 result to increases profit up to 25%.
- The profit will decrease noticeably due to increases in β and τ . The variation -20% to 20% in b , profit will be decreases significantly up to 45%. If the increases in cost parameters A, C, h, h_u then profit decreases moderately.
- Total profit slightly decreases due to increases in δ . The higher rate of deterioration θ of new product which affects gradually decreases the retailer's total profit.

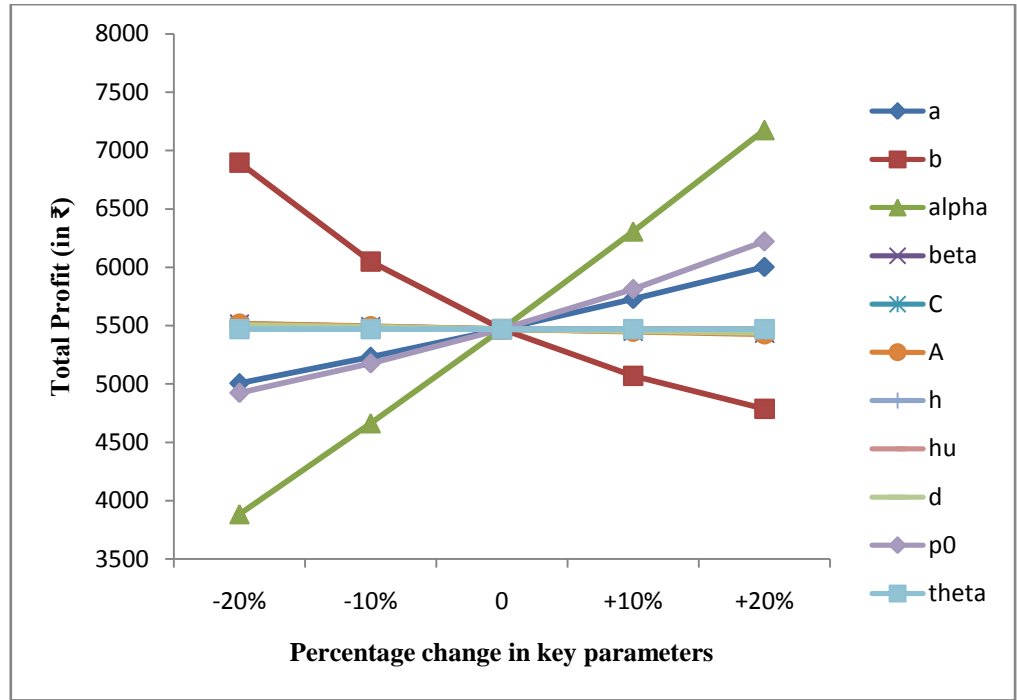


Figure 4.10 Effect of inventory parameters on total profit for model 4.2

- Optimal selling price increases up to 30% if a and α increases -20% to 20%. Increases in p_0 then optimal selling price extremely increases, observed that p_0 increases -20% to 20% then selling price massively increases up to 70%. If increases in $\beta, h, h_u, d_r, \tau, \varepsilon$ and θ result to optimal selling price increases marginally.
- Increases -20% to 20% in b then selling noticeably decreases up to 60%. Optimal selling price also significantly decreases with increases in C . Increases in θ_u and δ , selling price will be slightly decreases.

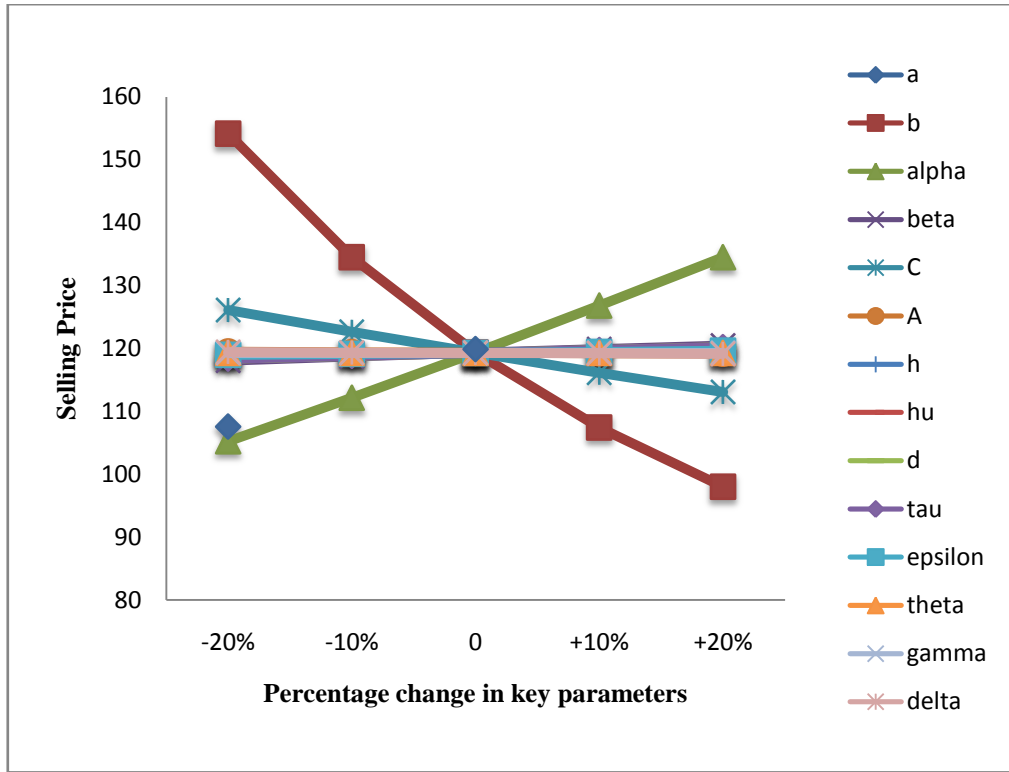


Figure 4.11 Effect of inventory parameters on selling price for model 4.2

- If increases b then positive inventory duration increases supremely. Positive inventory period increases slightly with increases in A, C, α, τ . On the other side, increases in remaining parameters then positive cycle time decreases moderately. When the values of parameters $A, \alpha, \tau, \epsilon, \beta$ and p_0 increase, the optimal shortages period will be increases, but parameter b, C, δ increases, the optimal shortages period decreases.
- Optimal ordering quantity of new products increases extremely with increases in a . If increases in C, A, τ ordering quantity of new products marginally increases. If β and p_0 increases then Q^* decreases noticeably and other remains parameters increases then Q^* a little decreases. Buyback quantity Q_u^* hugely increases with increases in b and α . Increases in C and A , increases Q_u^* slightly. Increases in a then Q^* decreases noticeably.

4.3 Analysis of deterioration effects on retailer's profit

In this section, we analyze the impact of a product's deterioration in the case of partial backlog shortages. The different constant rate of deterioration of products effects on the retailer's profit, ordering quantity and buyback quantity is mentioned in Table 4.5.

Table 4.5 Effects of product's deterioration on retailer's profit

Case(s):	θ	θ_u	Q^* (units)	Q_u^* (units)	Total Profit	Profit behaviour
Rate of deterioration is zero. (Model 4.1)	0	0	15.69	13.93	5471.14	↓
Rate of deterioration is same for both products	0.01	0.01	15.60	13.94	5470.69	
Rate of deterioration of used buyback product is higher than new products	0.01	0.012	15.60	13.94	5470.81	↑
Rate of deterioration of used buyback products is double than new products (Model 4.2)	0.01	0.02	15.61	13.96	5471.20	
Rate of deterioration of new products is higher than used products.	0.01	0.008	15.60	13.93	5470.61	↓
	0.01	0.005	15.60	13.92	5470.32	

4.4 Discussion about managerial insights

The recommended models are helpful to optimize for managerial insights. The prime insights are listed below.

- The optimum values of positive cycle time, shortage period, and selling price are helpful to retailers in deciding when to replenish orders, how much shortage period such that minimize backorder cost and lost sale cost, and how much to order new products at, which is useful to maintain market demand. The optimum buyback quantity and new product quantity are helpful in determining how much quantity of new products to replenish; as a result, the total profit is maximized from new products as well as used products.
- A higher, constant demand for new products motivates a retailer to establish a high selling price and increases profit. The buyback rate of used products and the corresponding demand of used products increase the retailer's profit, and it is also

beneficial to environmental protection because higher demand for used products reduces the need for raw materials for the production of new products.

- Retailer profit slightly reduces due to increase ordering cost, purchase cost and holding cost. This finding implies that the ordering cost, purchase cost and holding cost should be properly maintained by the retailer to increase their profit.
- According to our data, retailers who offer a possible maximum price discount on used products to buyers during resale, boost overall profit by raising the selling price. This finding implies that retailer gives to more price discount on used buyback product during resell to customers, increases total profit with increases selling price.
- The retailer's total profit eventually declines due to the increased rate of new product deterioration and selling price.
- Natural occurrences demonstrate that older items decay more quickly than newer ones. The analysis we conducted shows that if the rate of deterioration for old products is greater than for new products, the retailer will make a greater profit. The results of our study show that profit decreases if we believe that the rate of deterioration of older products is lower than that of newer products, but older products degrade at a slower rate than new ones, which is not always practicable.
- In the case of non-deteriorating products, the retailer's profit is higher compared to considering deteriorating products. It is obvious that the deterioration and preservation costs are minimal for the products whose deterioration rate is equivalent to zero.
- The minimum rate of depreciation on purchase cost for used buyback product is beneficial to retail for gain profit. Higher depreciation on purchase cost for used buyback product and higher selling price decreases the retailer's total profit. The retailer should maintain minimum depreciation on the purchase cost of buyback product.
- The negative impact on the total profit of retailer's for the higher value of backlogging parameter.
- The stock-out period should be a minimum from a profit point of view; a longer shortage period may increase backorders and lost sales. Our analysis shows that higher values of the backlog parameter, lost sale cost per unit, and backorder cost per unit decrease the retailer's profit. It indicated that the retailer should possibly replenish the order of new products as early as possible and increase the buyback rate of used products to satisfy demand as early as possible.

4.5 Conclusion

In this chapter, we analyzed retailer's twin mathematical models of inventory structures for products with and without deterioration. The retailer trades the new product as well as take-back used products from customers and resells them again. The demand can be satisfied by new products and buyback used products, unsatisfied demand is partially backlogged. We proposed two inventory models to maximize the total profit of the retailer by obtaining the optimal selling price, positive inventory period, shortages period, ordering quantity for new products, and optimal buyback quantity of used products using the classical optimization method. 'A bottle distributor for the beverage producer is one of illustration related to this chapter. Buyers' used bottles are taken back, cleaned, and sanitized before being combined with the stock of newly bought bottles. Choosing a strategy for the ordering quantity of new bottles, when to order new bottles and retrieving old bottles is the retailer's problem'. Some of the key innovative ideas discussed in this chapter are: (i) we hypothesize that demand is price-sensitive, time-dependent, and exponentially declining because time has an impact on demand, (ii) the effects of the deterioration rate of new and used products on the retailer's profit and quantity of products are analyzed, (iii) the impact of shortage periods, backlog rates, and price discounts on used buyback products is determined. Finally, numerical examples, graphical representations, and sensitivity analyses are provided to illustrate the proposed model. The managerial insights were derived as an outcome of the proposed study. 'Using preservation technology to reduce deterioration' can be an authentic extension of the proposed model. Another, scope for future study is 'the demand function to be considered a dependent, on advertisement, stock, or greening efforts'. Apart from this, a further study should include the influence of different payment systems to be employed.

CHAPTER-5

An EOQ Model for Deteriorating Products with Green Technology Investments and Trade Credit Payment System

5.1 Introduction

People's awareness of sustainability problems has grown over the past few decades as a consequence of global warming and an increase in catastrophic weather events. One of the factors contributing to global warming is the emission of carbon from numerous sources. In order to mitigate global warming, several nations, areas, and local governments have put in place regulations or programs for tracking emissions, like a carbon tax and carbon cap-and-trade, among others Toptal [23], Zhang [243]. This chapter considered the sources of carbon emissions from transportation and storing inventory processes. Both relate to energy use and emissions, which directly or indirectly increase carbon emissions Bonney and Jaber [170], chen et al. [171], Hasan et al [202], Battini et al. [244]. The amount of carbon emissions during logistical and storage activities might be reduced with an investment in green technologies Tao and Xu[201]. According to a recent survey of European Commission, 2013, 40.7% of customers said that the sustainability credentials of products or services had an impact on their choice of products to buy and demand from consumers who depend on investments in green technology benefits the environment by Zanoni et al. [197]. In light of these factors, consumer demand can be described as a linear function of the carbon reduction function, as a green investment and the selling price, which is ideal for perishable products with fixed expiration dates that are prone to deterioration and whose demand increases with investments in green technology and decreases with the selling price. The retailer is

permitted a credit period to acquire cash flow by the supplier, and hence, the retailer gets more time to invest in green technology and manage sustainability. According to carbon policies and trade credit financing, three cases are discussed: (1) Carbon tax policy with trade credit payment system; (2) Carbon cap-trade policy with trade credit payment system; and (3) Carbon tax and cap-trade policy except trade credit payment system. The objective of this chapter is to maximize the total profit of the retailer at the optimum replenishment cycle, green investment cost, and selling price by using the classical optimization method for each case. Various numerical examples and graphic representations of the objective function are shown to validate the resulting model. Subsequently, by performing sensitivity analysis on the decision variables and altering the inventory parameters, significant managerial insights are generated that are beneficial for the retailer. The analysis of the instances suggests that case 2, specifically sub-case 2.2, will yield the highest profit.

5.2 Notations and Assumptions

The structure of the suggested model includes the following notations and assumptions.

5.2.1 Notations

Parameters

A	Retailer ordering cost (in ₹/order)
C_0	Original purchase cost except transportation cost (Constant) (in ₹/unit)
C_T	The cost for transportation of product from supplier to the retailer. (Constant) (in ₹/unit)
h	Inventory holding cost (in ₹/unit)
e_h	Carbon emissions from holding operations per cycle (kilogram/unit.)
e_T	Carbon emission during delivery of inventory per cycle (kilogram/unit.)
Q	The replenishment quantity
$I(t)$	Inventory level at time, $0 \leq t \leq T$ (units)
ρ	Carbon tax (in ₹/ kilogram)
c_{cap}	Total carbon cap or limit (in ₹/ kilogram) in cap-trade carbon policy
m	The time to expiration date or the maximum shelf life in units of time,

$m > 0$	
d	Distance travelled by vehicle (in kilometre)
ω_1	Efficiency factor of Green technology
ω_2	Emission factor of green technology
I_e	Rate of interest earned (in percentage).
I_c	Rate of interest charged (percentage), where $I_c \geq I_e$
M	Allowable delay in the payment received from the buyer by the seller (years)
c_{tp}	Carbon trading price (in ₹/kilogram) for cap-trade policy
\hat{C}	Total carbon emissions before investing in green technology (kilogram/unit)
\hat{C}_g	Total carbon emissions after investing green technology (kilogram/unit).

Decision variables

g	Green technology investment cost (in ₹/ unit/cycle)
p	Selling Price (in ₹/unit)
T	Replenishment cycle time (in year)

Expressions and functions

$f(g)$	Carbon reduction function depending on green investment
$R(g, p)$	Demand function depending on green investment cost and selling price
$\theta(t)$	Deterioration rate as a time-varying function ($0 \leq \theta(t) \leq 1$)
$I(t)$	Inventory level at time t , $0 \leq t \leq T$
$\pi(g, p, T)$	Total profit except for trade credit and carbon policies.
$\pi_1(g, p, T)$	Total profit for $M < T$ except carbon policies
$\pi_2(g, p, T)$	Total profit for $M \geq T$ except carbon policies

Objective functions

$TP_1(g, p, T)$	Total profit for sub-case 1.1
$TP_2(g, p, T)$	Total profit for sub-case 1.2
$TP_3(g, p, T)$	Total profit for sub-case 2.1
$TP_4(g, p, T)$	Total profit for sub-case 2.2
$TP_5(g, p, T)$	Total profit for sub-case 3.1

$TP_6(g, p, T)$ Total profit for sub-case 3.2

5.2.2 Assumptions

1. An EOQ model constructed for single type of product.
2. Replenishment rate is instantaneous and planning horizon to be considered infinite.
3. Cost for holding inventory in warehouse is constant.
4. The rate of deterioration of perishable product is time dependent and it is defined as

$$\theta(t) = \frac{1}{1+m-t}, 0 \leq t \leq T \leq m, \text{ where } m \text{ is expiration date. All perishable goods start to}$$

gradually deteriorate and become unable to be sold whenever the expiration date m passes.
5. Emissions of carbon in the environment are caused due to delivery of inventory and the inventory holding process in storage. (Bonney and Jaber [170] , Chen et al.[171], Hasan et. al [202])
6. The protection of ecosystems benefits from investments in green technologies. Therefore, we treated the function of reducing carbon emissions as a green investment cost. The carbon reduction function in form of green investment is $f(g) = \omega_1 g - \omega_2 g^2$, where $g < \frac{\omega_1}{\omega_2}$, $\omega_1 > 0$ denotes efficiency factor of green technology and $\omega_2 > 0$ denotes emission factor of green technology.(Huang and Rust[175],Toptal et al.[23] and Hasan et al.[202])
7. The sustainability affect positive on buyer's decisions and investment in green technology impact positively on environment and buyer's demand (Zanoni et al.[197] , Hasan et. al.[202]) In our study demand function to be considered depends on green technology investment as a carbon reduction function and selling price dependent. Defined as $R(g, p) = \alpha + \beta f(g) - \gamma p$, where $\alpha > 0$ denotes the scale demand, $\beta > 0$ is constant coefficient of $f(g)$ and $0 < \gamma < 1$ denotes the price elasticity.
8. The retailer invests in green technology for sustainability over a certain time frame without raising the unit price of the product.
9. Carbon taxation and Carbon cap-trade policies applied.
10. The Lead time is negligible or zero and shortages are not allowed.

11. According to trade credit payment policy, from the consumer payments, the retailer can earn the interest with interest rate I_e , per unit per year because retailer need not to pay any amount till M . At time M account will be settle by retailer, M is a permissible delay duration offered by the supplier to retailer. After time M supplier will be charge the interest to the retailer on the unsold inventory with rate I_c . (Soni et al.[111], Shah and Jani[116], Taleizadeh et al.[190],Soni [245], Sarkar[73]).

5.3 Mathematical formulation

In this section, we formulated the inventory modelling with considering carbon policies and permissible delay payment strategy. According to notations and assumptions, the status of inventory $I(t)$ of perishable products at time t during the replenishment cycle $[0, T]$ is depleted by join effects of demand and deterioration, consequently, the inventory level at time t is governed by the following differential equation,

$$I'(t) + \theta(t)I(t) = -R(g, p), 0 \leq t \leq T \leq m \quad (5.1)$$

Solving (5.1) with boundary condition $I(T) = 0$ yields,

The solution of the differential equation (5.1) is given by,

$$I(t) = R(g, p)(1 + m - t) \ln \left(\frac{1 + m - t}{1 + m - T} \right), 0 \leq t \leq T \leq m \quad (5.2)$$

So, the ordering quantity per cycle time T is as follows,

$$Q = I(0) = R(g, p)(1 + m) \ln \left(\frac{1 + m}{1 + m - T} \right) \quad (5.3)$$

Transportation and inventory storage operations of inventory are the sources of carbon emissions. The distance travelled by vehicle is one of the roles to emitted carbon, in our study the carbon emissions for shipping is measured by $e_t \frac{R(g, p)}{Q} d$ (Hasan et.al [202],

Huang et al.[191]) and carbon emission during the storage operations for a certain time frame is calculated as $e_h \int_0^T I(t) dt$.

Total carbon emission from delivery of inventory from supplier to retailer and emission from inventory storage process is,

$$\hat{C} = e_T \frac{R(g, p)}{Q} d + e_h \int_0^T I(t) dt \quad (5.4)$$

By investing a particular amount in green technology is g , the retailer reduces the per-unit carbon emissions. The emission reduction function is $f(g) = \omega_1 g - \omega_2 g^2$, where $g < \frac{\omega_1}{\omega_2}$.

Hence, the total carbon emission after investing in green technology is

$$\begin{aligned} \hat{C}_g &= e_T \frac{R(g, p)}{Q} d + e_h \int_0^T I(t) dt - f(g) \\ &= e_T \frac{(\alpha + \beta(\omega_1 g - \omega_2 g^2) - \gamma p)}{Q} d + e_h \int_0^T I(t) dt - (\omega_1 g - \omega_2 g^2) \end{aligned} \quad (5.5)$$

Now to calculate total profit, we calculate all the factors as below,

Sales revenue from the selling the product :

$$SR = p \left(\int_0^T (\alpha + \beta f(g) - \gamma p) dt \right) \quad (5.6)$$

$$\text{Product's cost of ordering: } OC = A \quad (5.7)$$

$$\text{The cost of holding operations: } HC = h \int_0^T [I(t)] dt \quad (5.8)$$

$$\text{The total purchase cost (including transporting cost): } PC = (C_0 + C_T)Q \quad (5.9)$$

Investment cost in green technology:

$$GTC = g \quad (5.10)$$

Total profit of retailer from (5.6) to (5.10) can be written as,

$$\pi(g, p, T) = SR - OC - HC - PC - GTC \quad (5.11)$$

Taking into consideration the credit period given by supplier to the retailer is M , the two possibilities arose, $M < T$ and $M \geq T$. These are explained as below,

Possibility 1: $M < T$

After the time M , the retailer needs to pay the interest for the items in stock. Hence, the interest charged per cycle is as follows,

$$IC_1 = C_0 I_c \int_M^T I(t) dt \quad (5.12)$$

Before the time M , revenue can be accumulated by the retailer as,

$$IE_1 = p I_e \int_0^M R(g, p) t dt \quad (5.13)$$

Therefore, the total profit after applying the trade credit policy is obtained per year is,

$$\pi_1(g, p, T) = SR - OC - HC - PC - GTC - IC_1 + IE_1 \quad (5.14)$$

Possibility 2: $M \geq T$

The replenishment cycle time T is shorter than or equal to the credit period M . In this sub-case, all items are sold during the settlement; the retailer accumulates interest at time T . The supplier charges no interest, and the interest accumulated per order during in time M is obtained as,

$$IE_2 = p I_e \left[\int_0^T R(g, p) t dt + R(g, p) T (M - T) \right] \text{ and} \quad (5.15)$$

$$IC_2 = 0 \quad (5.16)$$

Therefore, the total profit for this possibility is obtained per year is,

$$\pi_2(g, p, T) = SR - OC - HC - PC - GTC - IC_2 + IE_2 \quad (5.17)$$

Next, the case of credit payment is discuss for a carbon tax and cap-trade policy, then carbon tax and cap-trade policy are analyzed except credit payment policy.

Case 1 Carbon tax policy with trade credit payment system:

Under a carbon tax policy, the government or authorities sets a price that any emitters must pay for each unit of greenhouse gas emissions they emit. Businesses and customers will take steps, including switching fuels or adopting new technologies, to lessen their emissions to keep away from paying the extra tax. In the case of carbon tax and trade credit payment policy with green investing operations, there are two sub-cases discussed as below,

Sub-case 1.1 Carbon tax policy and $M < T$

After applying carbon tax policy and investing in green technology, the total profit of retailer per year for the sub-case $M < T$ is,

$$\begin{aligned}
TP_1(g, p, T) &= \frac{1}{T} \left(\pi_1(g, p, T) - \rho \hat{C}_g \right) \\
&= \frac{1}{T} \left(p \int_0^T R(g, p) dt - A - h \int_0^T [I(t)] dt - (C_0 + C_T)Q - g - C_0 I_c \int_M^T I(t) dt \right. \\
&\quad \left. + p I_e \int_0^M R(g, p) t dt - \rho \left(e_T \frac{R(g, p)}{Q} d + e_h \int_0^T I(t) dt - (\omega_1 g - \omega_2 g^2) \right) \right) \quad (5.18)
\end{aligned}$$

Sub-case 1.2 Carbon tax policy and $M \geq T$

According to the carbon tax policy and investing in green technology, the total profit of retailer per year for the sub-case $M \geq T$ is,

$$\begin{aligned}
TP_2(g, p, T) &= \frac{1}{T} \left(\pi_2(g, p, T) - \rho \hat{C}_g \right) \\
&= \frac{1}{T} \left(p \int_0^T R(g, p) dt - A - h \int_0^T [I(t)] dt - (C_0 + C_T)Q - g \right. \\
&\quad \left. + p I_e \left[\int_0^T R(g, p) t dt + R(g, p) T(M - T) \right] \right. \\
&\quad \left. - \rho \left(\frac{R(g, p)}{Q} e_T d + e_h \int_0^T I(t) dt - (\omega_1 g - \omega_2 g^2) \right) \right) \quad (5.19)
\end{aligned}$$

Case 2 Carbon cap-trade policy with trade credit payment system:

The total carbon emissions cost under the cap and trade policy is $Cap^c = c_{tp}(\hat{C}_g - c_{cap})$. In the case of carbon cap-trade policy and trade credit payment system, there are two sub-cases discussed as below,

Sub-case 2.1 Carbon cap-trade policy and $M < T$

Total profits after applying carbon cap and trade regulation for $M < T$ during the cycle time is,

$$TP_3(g, p, T) = \frac{1}{T} \left(\pi_1(g, p, T) - Cap^c \right) \quad (5.20)$$

$$= \frac{1}{T} \left(p \int_0^T R(g, p) dt - A - h \int_0^T [I(t)] dt - (C_0 + C_T)Q - g - C_0 I_c \int_M^T I(t) dt \right. \\ \left. + p I_e \int_0^M R(g, p) t dt - c_{tp} \left(\left(e_T \frac{R(g, p)}{Q} d + e_h \int_0^T I(t) dt - (\omega_1 g - \omega_2 g^2) \right) - c_{cap} \right) \right)$$

Sub-case 2.2 Carbon cap-trade policy and $M \geq T$

Total profits after applying carbon cap-trade regulation for $M \geq T$ is

$$TP_4(g, p, T) = \frac{1}{T} (\pi_2(g, p, T) - Cap^c) \quad (5.21)$$

$$= \frac{1}{T} \left(p \int_0^T R(g, p) dt - A - h \int_0^T [I(t)] dt - (C_0 + C_T)Q - g \right. \\ \left. + p I_e \left[\int_0^M R(g, p) t dt + R(g, p) T (M - T) \right] \right. \\ \left. - c_{tp} \left(\left(e_T \frac{R(g, p)}{Q} d + e_h \int_0^T I(t) dt - (\omega_1 g - \omega_2 g^2) \right) - c_{cap} \right) \right)$$

Case 3 Carbon policies except trade credit payment system:

The retailer's total profit obtains for both carbon policies without considering the trade credit payment system in this case.

Sub-case 3.1 Total profits under carbon tax regulation

$$TP_5(g, p, T) = \frac{1}{T} (\pi(g, p, T) - \rho \hat{C}_g)$$

$$= \frac{1}{T} \left(\left(p \int_0^T R(g, p) dt - A - h \int_0^T [I(t)] dt - (C_0 + C_T)Q - g \right) \right. \\ \left. - \rho \left(\frac{R(g, p)}{Q} e_T d + e_h \int_0^T I(t) dt - (\omega_1 g - \omega_2 g^2) \right) \right) \quad (5.22)$$

Sub-case 3.2 Total profits under carbon cap-trade regulation

$$TP_6(g, p, T) = \frac{1}{T} (\pi(g, p, T) - Cap^c) \quad (5.23)$$

$$= \frac{1}{T} \left(\begin{array}{c} \left(p \int_0^T R(g, p) dt - A - h \int_0^T [I(t)] dt - (C_0 + C_T)Q - g \right) \\ -c_p \left(e_T \frac{R(g, p)}{Q} d + e_h \int_0^T I(t) dt - (\omega_1 g - \omega_2 g^2) - c_{cap} \right) \end{array} \right)$$

The total profit function is a function of green technology investment g , selling price p and the replenishment cycle time T . The objective is to find the optimal value of green technology investment; selling price and the replenishment cycle time such that the retailer's total profit $TP_i(g, p, T)$, $i = 1, 2, 3, 4, 5, 6$ is maximized.

5.3.1 Solution technique to determine the optimal solution

The necessary conditions for maximize of the total profit function given by (5.18) to (5.23) are,

$$\frac{TP_i(g, p, T)}{\partial g} = 0, \frac{TP_i(g, p, T)}{\partial p} = 0, \frac{TP_i(g, p, T)}{\partial T} = 0, i = 1, 2, 3, 4, 5, 6 \quad (5.24)$$

To check the sufficient condition for maximize $TP_i(g, p, T)$, $i = 1, 2, 3, 4, 5, 6$ by using the Hessian matrix method, consider Hessian matrix,

$$H_i(g, p, T) = \begin{pmatrix} \frac{\partial^2 TP_i(g, p, T)}{\partial g^2} & \frac{\partial^2 TP_i(g, p, T)}{\partial g \partial p} & \frac{\partial^2 TP_i(g, p, T)}{\partial g \partial T} \\ \frac{\partial^2 TP_i(g, p, T)}{\partial p \partial g} & \frac{\partial^2 TP_i(g, p, T)}{\partial p^2} & \frac{\partial^2 TP_i(g, p, T)}{\partial p \partial T} \\ \frac{\partial^2 TP_i(g, p, T)}{\partial T \partial g} & \frac{\partial^2 TP_i(g, p, T)}{\partial T \partial p} & \frac{\partial^2 TP_i(g, p, T)}{\partial T^2} \end{pmatrix} \quad (5.25)$$

Define that $\Delta_{11} = \frac{\partial^2 TP_i}{\partial g^2}$, $\Delta_{22} = \frac{\partial^2 TP_i}{\partial g^2} \cdot \frac{\partial^2 TP_i}{\partial p^2} - \left(\frac{\partial^2 TP_i}{\partial g \partial p} \right)^2$ and $\Delta_{33} = \det(H_i)$.

So, the sufficient conditions are $\Delta_{11} < 0$, $\Delta_{22} > 0$ and $\Delta_{33} < 0$ (5.26)

We verify the sufficient conditions at the optimum value of decision variables for each case. The corresponding optimization problem is nonlinear in nature. So we prefer numerical as well as graphical way of representing the solution helping us the visualization of the concave nature of average total profit function.

For determine the optimum value of decision variables and to identify the concave nature of the total profit at the optimal solution, applying the solution procedure for each sub-case as per following steps:

Step 1: First allocated proper hypothetical values to the inventory parameters in $TP_i(g, p, T), i = 1, 2, 3, 4, 5, 6$.

Step 2: Solving the simultaneous equations stated in (5.24) using the mathematical software like, Maple XVIII or mathematica or matlab, to find the value of decision variable g, p, T .

Step 3: Find the hessian matrix at optimal value of g^*, p^*, T^* and check the sufficient conditions for maxima from (5.25).

Step 4: Substitute the corresponding optimal value of g^*, p^*, T^* in (5.18) to (5.23) and obtained total profit.

Step 5: From (5.3), obtain optimal value of replenishment order quantity.

5.4 Numerical experiment

Consider the following example to see how the proposed model works.

Example 5.4.1: The numerical, graphical, and sensitivity analyses of the model used the following numerical values of the parameter in correct units as input. Here, real-world data is taken into account, with some modifications made in accordance with the developed model. Input value of each parameter taken as per the table below with units of parameters described in notations section:

Table 5.1 Input parameters of proposed model

Input parameters			
Common Inputs (CI)	$\omega_1 = 5, \omega_2 = 0.5, A = 100, m = 2, d = 90, C_0 = 150, C_T = 1, e_h = 0.2, e_r = 0.3, h = 0.7, \alpha = 30, \beta = 10, \gamma = 0.7$		
Case-1	Sub-case 1.1 carbon tax	$M < T$	$CI, \rho = 15, M = \frac{30}{365}, I_e = 10\%, I_c = 15\%$
	Sub-case 1.2 carbon tax	$M \geq T$	$CI, \rho = 15, M = \frac{300}{365}, I_e = 10\%, I_c = 15\%$
Case-2	Sub-case 2.1 cap-trade	$M < T$	$CI, c_{tp} = 12, c_{cap} = 4, M = \frac{30}{365}, I_e = 10\%, I_c = 15\%$

	Sub-case 2.1 cap-trade	$M \geq T$	$CI, c_{ip} = 12, c_{cap} = 4, M = \frac{300}{365}, I_e = 10\%,$ $I_c = 15\%$
Case-3	Sub-case 3.1 carbon tax	except trade	CI and $\rho = 15$
	Sub-case 3.1 cap-trade	credit policy	$CI, c_{ip} = 12, c_{cap} = 4$

Applying the solution procedure described in section 5.3.1, the optimal results are found as mentioned in Table 5.2.

Table 5.2 Optimum results of model by numerical experiment

Case	Sub-case	g^* (in ₹)	T^* (Year)	p^* (in ₹)	Q^* (Units)	\hat{C}_g^*	TP_i (in ₹)	Carbon Reduced
1	1.1	4.999	0.6818	345.52	40	25	8144.51	12
	1.2	4.999	0.6313	339.62	38	28	9105.24	12
2	2.1	4.999	0.6356	344.43	37	27	8336.71	12
	2.2	4.999	0.5860	338.58	35	31	9328.49	12
3	3.1	4.999	0.7103	345.21	42	24	8231.72	12
	3.2	4.999	0.6614	344.18	39	26	8410.17	12

The concavity of the profit function is evolved by using the hessian matrix for each case as below; from (5.25), hessian matrix at optimum value for sub-case 1.1 is,

$$H_1(g^*, p^*, T^*) = \begin{pmatrix} -1734.282891 & 0.00846075921 & 1.755778221 \\ 0.00846075921 & -0.6002972285 & 13.0125595 \\ 1.755778221 & 13.0125595 & -11803.2781 \end{pmatrix} \quad (5.27)$$

Similarly, we can obtain the hessian matrix as per (5.25) for all others remaining sub-cases 1.2 to 3.2 at g^*, p^*, T^* . The sufficient conditions using hessian matrix method for all sub-cases are verified through numerically at g^*, p^*, T^* as in following table;

Table 5.3 Validation of sufficient conditions

Case	Sub-case	Δ_{11}	Δ_{22}	Δ_{33}
1	1.1	$-1734.28 < 0$	$1441.09 > 0$	$-1.20 \times 10^7 < 0$
	1.2	$-1883.86 < 0$	$1187.54 > 0$	$-1.77 \times 10^8 < 0$
2	2.1	$-1742.13 < 0$	$1045.84 > 0$	$-1.25 \times 10^7 < 0$
	2.2	$-1895.55 < 0$	$1797.48 > 0$	$-1.89 \times 10^7 < 0$

3	3.1	$-1735.67 < 0$	$1041.40 > 0$	$-9.98 \times 10^7 < 0$
	3.2	$-1742.99 < 0$	$1045.79 > 0$	$-1.04 \times 10^7 < 0$

5.4.1 Graphical authentication of the concavity of objective functions

The graphical validations of the concavity of the objective functions of the proposed model derived as per below figures, which indicated that the total profit is maximized with respect to the decision variables.

The concavity behaviour of profit function $TP_1(g, p, t)$ for the sub-case 1.1 is shown in Figure 5.1 and Figure 5.2.

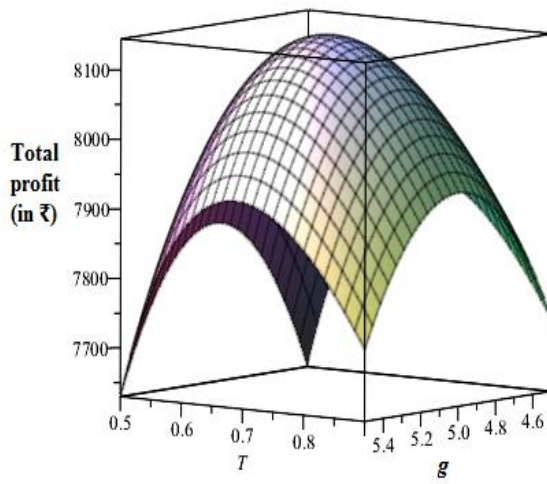


Figure 5.1 Concavity of $TP_1(g, p, t)$ with respect to T and g .

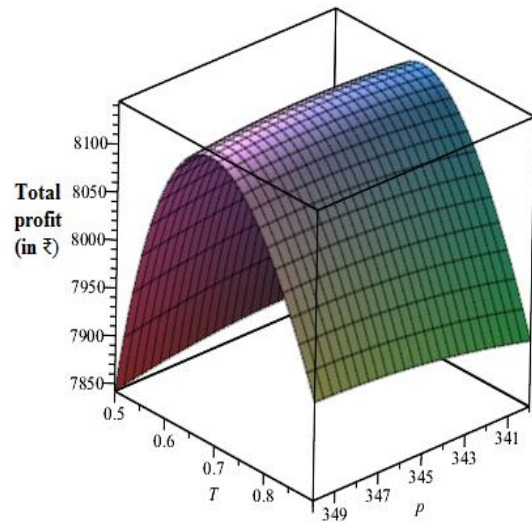


Figure 5.2 Concavity of $TP_1(g, p, t)$ with respect to T and p .

The concavity behaviour of profit function $TP_2(g, p, t)$ for the sub-case 1.2 is shown in Figure 5.3 and Figure 5.4.

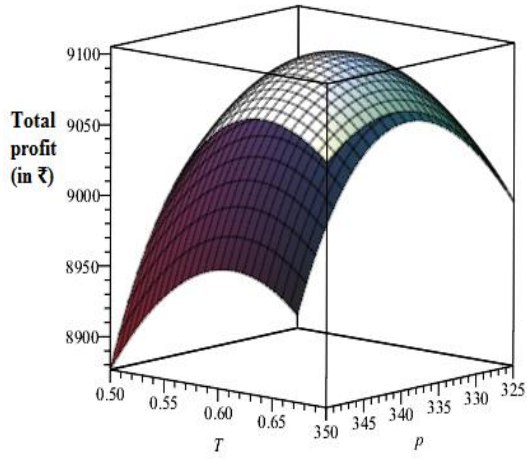


Figure 5.3 Concavity of $TP_2(g, p, t)$ with respect to T and p

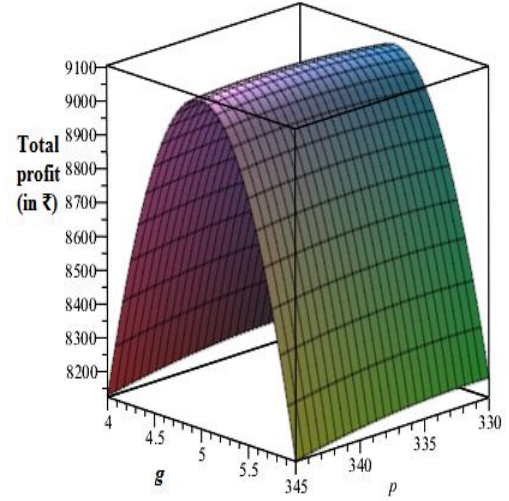


Figure 5.4 Concavity of $TP_2(g, p, t)$ with respect to g and p

The concavity behaviour of profit function $TP_3(g, p, t)$ for the sub-case 2.1 is shown in Figure 5.5 and Figure 5.6.

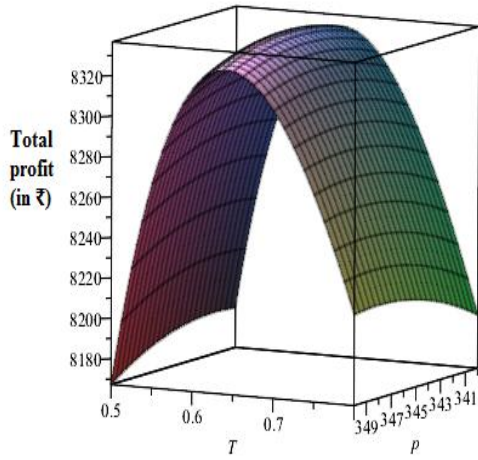


Figure 5.5 Concavity of $TP_3(g, p, t)$ with respect to T and p

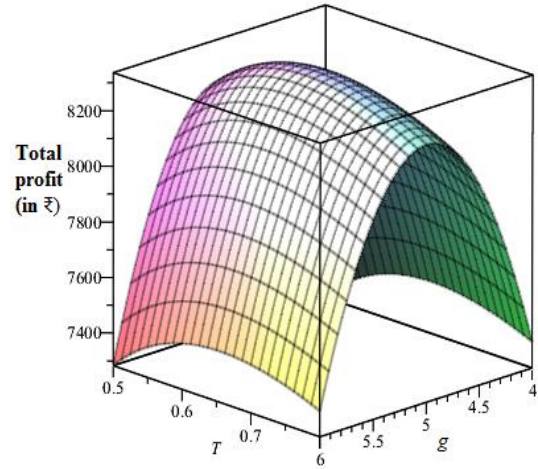


Figure 5.6 Concavity of $TP_3(g, p, t)$ with respect to T and g

The concavity behaviour of profit function $TP_4(g, p, t)$ for the sub-case 2.2 is shown in Figure 5.7 and Figure 5.8.

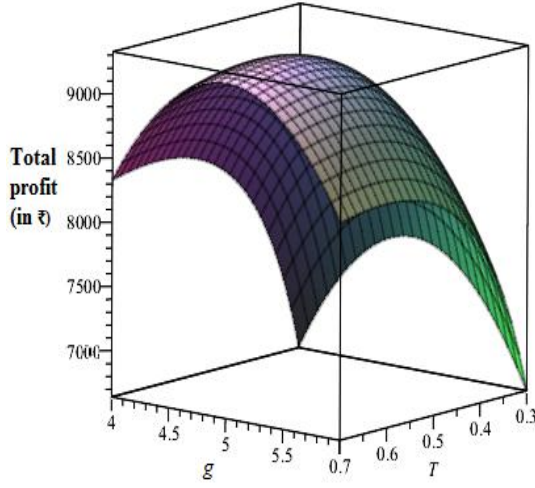


Figure 5.7 Concavity of $TP_4(g, p, t)$ with respect to g and T

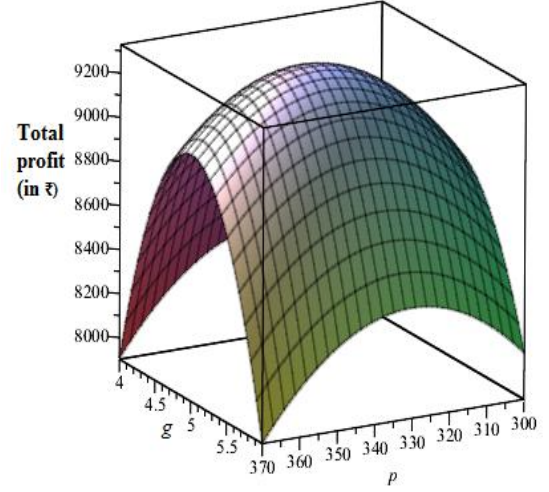


Figure 5.8 Concavity of $TP_4(g, p, t)$ with respect to g and p

The concavity behaviour of profit function $TP_5(g, p, t)$ for the sub-case 3.1 is shown in Figure 5.9 and Figure 5.10.

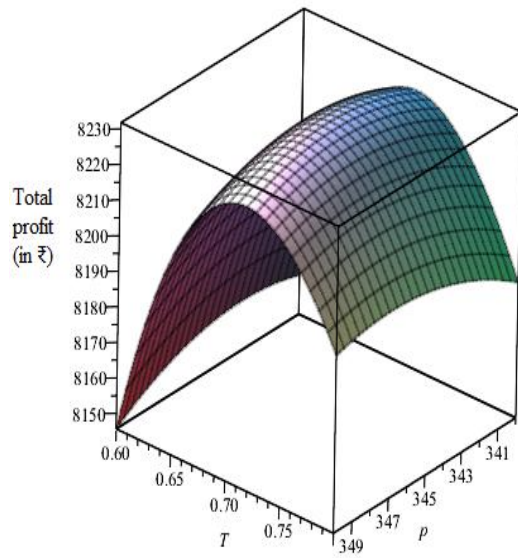


Figure 5.9 Concavity of $TP_5(g, p, t)$ with respect to T and p

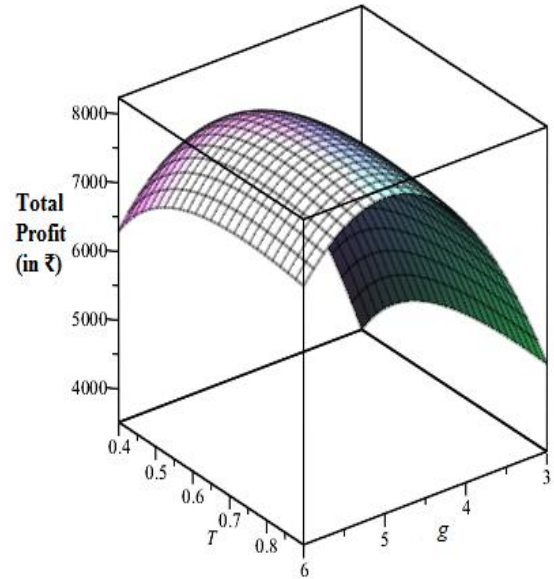


Figure 5.10 Concavity of $TP_5(g, p, t)$ with respect to T and g

The concavity behaviour of profit function $TP_6(g, p, t)$ for the sub-case 3.2 is shown in Figure 5.11 and Figure 5.12.

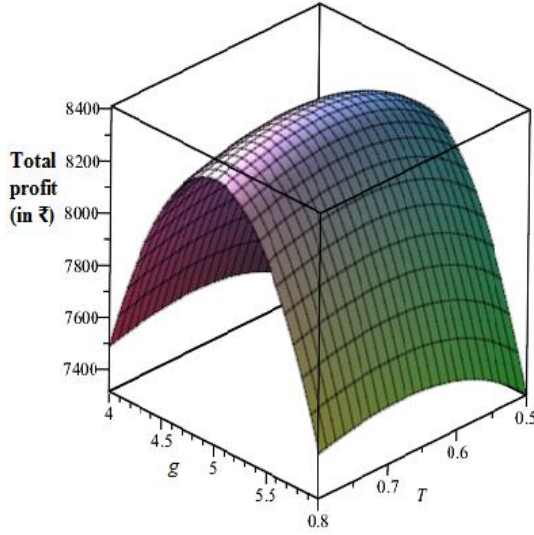


Figure 5.11 Concavity of $TP_6(g, p, t)$ with respect to g and T

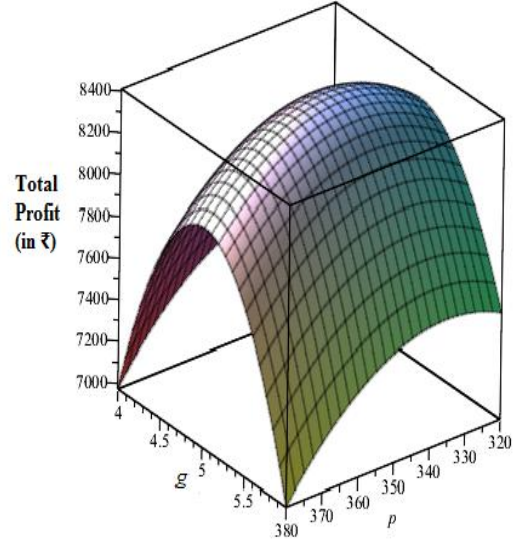


Figure 5.12 Concavity of $TP_6(g, p, t)$ with respect to g and p

5.5 Sensitivity analysis and observations

We now analyze the effect of changes in system key parameters on the optimal values base on numerical examples taken in section 5.4, sensitivity analysis is performed by changing each parameter values in relative steps of -20% , -10% , $+10\%$, $+20\%$, taking one parameter at a time and the remaining values of the parameters are unchanged. The findings are displayed in Table 5.4, Table 5.5 and Table 5.6 for the case-1, case-2 and case-3 respectively.

Table 5.4 Sensitivity effects of parameters in optimal results for case-1

Parameters	Value	$M < T$					$M \geq T$				
		g^* (in ₹)	T^* (Year)	p^* (in ₹)	Q^* (Unit)	Profit (in ₹)	g^* (in ₹)	T^* (Year)	p^* (in ₹)	Q^* (Unit)	Profit (in ₹)
α	24	4.999	0.6937	335.78	38	7147.92	4.999	0.6443	329.90	36	8021.90
	27	4.999	0.6876	340.65	39	7638.52	4.999	0.6377	334.76	37	8555.39
	33	4.999	0.6763	350.4	41	8665.88	4.999	0.6251	344.49	38	9671.44
	36	4.999	0.6709	355.29	41	9202.62	4.999	0.6191	349.36	39	10253.98
β	8	4.999	0.7384	305.11	33	4399.54	4.999	0.693	299.29	32	5024.05
	9	4.999	0.7076	325.25	36	6137.85	4.999	0.6596	319.39	35	6922.19
	11	4.999	0.6599	365.88	43	10418.2	4.999	0.6068	359.94	40	11572.15
	12	4.999	0.6407	386.31	46	12957.94	4.999	0.5852	380.32	43	14322.16
γ	0.24	4.999	0.6632	409.7	42	12698.04	4.999	0.6039	403.62	39	13953.65
	0.27	4.999	0.6722	374.02	41	10142.53	4.999	0.6178	368.04	38	11235.58
	0.33	4.999	0.6922	322.26	38	6552.29	4.999	0.6447	316.42	37	7402.54
	0.36	4.999	0.7033	302.94	37	5264.81	4.999	0.6582	297.14	36	6020.8

Parameters	Value	$M < T$					$M \geq T$				
		g^* (in ₹)	T^* (Year)	p^* (in ₹)	Q^* (Unit)	Profit (in ₹)	g^* (in ₹)	T^* (Year)	p^* (in ₹)	Q^* (Unit)	Profit (in ₹)
ω_1	4	3.999	0.8222	273.72	27	2099.72	3.999	0.7796	267.91	27	2497.6
	4.5	4.499	0.7427	307.29	33	4513.71	4.499	0.6965	301.45	32	5150.32
	5.5	5.499	0.6326	388.22	46	11127.01	5.499	0.5776	382.24	43	14679.7
	6	5.999	0.5912	435.28	52	20271.33	5.999	0.532	429.23	48	22226.1
ω_2	0.4	6.249	0.6247	396.39	47	14399.97	6.249	0.5689	390.4	44	15882.9
	0.45	5.555	0.6541	368.08	43	10718.18	5.555	0.6011	362.13	40	11897.4
	0.55	4.545	0.7084	327.16	37	6285.74	4.545	0.6599	321.29	35	7083.7
	0.6	4.166	0.7339	311.95	34	4907.43	4.166	0.6872	306.11	33	5580.57
ρ	12	4.999	0.6455	344.64	38	8261.77	4.999	0.5949	338.76	35	9247.19
	13.5	4.999	0.6645	345.09	39	8201.23	4.999	0.6138	339.2	37	9244.54
	16.5	4.999	0.6979	345.93	41	8091.01	4.999	0.6474	340.02	39	9040.96
	18	4.999	0.7128	346.32	42	8040.27	4.999	0.6625	340.39	40	8980.26
e_h	0.16	4.999	0.6833	345.44	40	8155.99	4.999	0.6325	339.55	38	9116.15
	0.18	4.999	0.6826	345.48	40	8150.25	4.999	0.6319	339.58	38	9110.69
	0.22	4.999	0.6811	345.56	40	8138.78	4.999	0.6307	339.66	38	9099.80
	0.24	4.999	0.6804	345.60	40	8133.06	4.999	0.6301	339.69	38	9094.36
e_T	0.24	4.999	0.6363	344.55	37	8309.51	4.999	0.5868	338.69	35	9300.47
	0.27	4.999	0.6601	345.05	38	8223.98	4.999	0.6100	339.17	36	9199.08
	0.33	4.999	0.7020	345.96	41	8070.11	4.999	0.6511	340.04	39	9017.69
	0.36	4.999	0.7208	346.38	42	8000.01	4.999	0.6696	340.44	40	8935.47
m	1.6	4.999	0.6499	346.64	38	7970.61	4.999	0.6066	340.65	36	8945.92
	1.8	4.999	0.6665	346.05	39	8063.25	4.999	0.6195	340.10	37	9030.99
	2.2	4.999	0.6961	345.06	40	8216.40	4.999	0.6421	339.19	38	9170.63
	2.4	4.999	0.7095	344.65	41	8280.46	4.999	0.6520	338.81	39	9228.66
d	72	4.999	0.6363	344.55	37	8309.51	4.999	0.5868	338.69	35	9300.47
	81	4.999	0.6601	345.05	38	8223.98	4.999	0.6100	339.17	36	9199.08
	99	4.999	0.7020	345.96	41	8070.11	4.999	0.6511	340.04	39	9017.69
	108	4.999	0.7208	346.38	42	8000.01	4.999	0.6696	340.44	40	8935.47
I_c	0.12	4.999	0.6868	345.45	40	8162.99	4.999	0.6313	339.62	38	9105.24
	0.135	4.999	0.6843	345.49	40	8153.68	4.999	0.6313	339.62	38	9105.24
	0.165	4.999	0.6794	345.56	40	8135.56	4.999	0.6313	339.62	38	9105.24
	0.18	4.999	0.6771	345.60	40	8126.31	4.999	0.6313	339.62	38	9105.24
I_e	0.08	4.999	0.6821	345.54	40	8142.75	4.999	0.6446	340.69	38	8924.04
	0.09	4.999	0.6820	345.53	40	8143.63	4.999	0.6378	340.15	38	9014.28
	0.11	4.999	0.6817	345.52	40	8145.40	4.999	0.6250	339.09	37	9196.92
	0.12	4.999	0.6816	345.51	40	8146.27	4.999	0.6190	338.57	37	9289.28

Table 5.5 Sensitivity effect of major parameters in decision results for case-2

Parameters	Value	$M < T$					$M \geq T$				
		g^* (in ₹)	T^* (Year)	p^* (in ₹)	Q^* (Unit)	Profit (in ₹)	g^* (in ₹)	T^* (Year)	p^* (in ₹)	Q^* (Unit)	Profit (in ₹)
c_{ip}	9.6	4.999	0.6022	343.67	35	8432.00	4.999	0.5527	337.83	33	9446.96
	10.8	4.999	0.6196	344.06	36	8382.68	4.999	0.5700	338.22	34	9385.51
	13.2	4.999	0.6504	344.78	38	8293.55	4.999	0.6008	338.92	36	9275.19
	14.4	4.999	0.6641	345.12	39	8252.81	4.999	0.6146	339.25	37	9225.07
	3.2	4.999	0.6376	344.47	37	8321.63	4.999	0.5878	338.62	35	9312.13
	3.6	4.999	0.6366	344.45	37	8329.16	4.999	0.5869	338.60	35	9320.31

Parameters	Value	$M < T$					$M \geq T$				
		g^* (in ₹)	T^* (Year)	p^* (in ₹)	Q^* (Unit)	Profit (in ₹)	g^* (in ₹)	T^* (Year)	p^* (in ₹)	Q^* (Unit)	Profit (in ₹)
c_{cap}	4.4	4.999	0.6346	344.41	37	8344.27	4.999	0.5851	338.56	35	9336.69
	4.8	4.999	0.6336	344.39	37	8351.83	4.999	0.5842	338.54	35	9344.90
M	0.0657	4.999	0.6360	344.47	37	8331.91	Not applicable				
	0.0739	4.999	0.6358	344.45	37	8334.18					
	0.0904	4.999	0.6353	344.41	37	8339.48					
	0.0986	4.999	0.6350	344.38	37	8342.51					
M	0.6575	Not applicable					4.999	0.5870	339.87	35	9031.65
	0.7397						4.999	0.5865	339.22	35	9179.94
	0.9041						4.999	0.5855	337.95	35	9477.29
	0.9863						4.999	0.5850	337.32	35	9626.34

Table 5.6 Sensitivity effect of major parameters in decision results for case-3

Parameters	Value	Carbon tax					Carbon cap-trade				
		g^* (in ₹)	T^* (Year)	p^* (in ₹)	Q^* (Unit)	Profit (in ₹)	g^* (in ₹)	T^* (Year)	p^* (in ₹)	Q^* (Unit)	Profit (in ₹)
ρ	12	4.999	0.6723	344.38	39	8338.18	Not applicable				
	13.5	4.999	0.6921	344.81	40	8283.22					
	16.5	4.999	0.7270	345.59	43	8183.15					
	18	4.999	0.7426	345.96	44	8137.06					
c_{tp}	9.6	Not applicable					4.999	0.6265	343.47	37	8496.10
	10.8						4.999	0.6447	343.83	38	8451.62
	13.2						4.999	0.6767	344.51	40	8371.27
	14.4						4.999	0.6910	344.82	41	8334.56
c_{cap}	3.2	Not applicable					4.999	0.6635	344.22	39	8395.67
	3.6						4.999	0.6624	344.20	39	8402.91
	4.4						4.999	0.6602	344.16	39	8417.42
	4.8						4.999	0.6591	344.14	39	8424.70
A	80	4.999	0.7061	345.13	41	8259.96	4.999	0.6568	344.10	38	8440.51
	90	4.999	0.7082	345.17	42	8245.82	4.999	0.6591	344.14	39	8425.31
	110	4.999	0.7124	345.25	42	8217.66	4.999	0.6636	344.22	39	8395.07
	120	4.999	0.7144	345.29	42	8203.65	4.999	0.6659	344.27	39	8380.03
C_T	0.8	4.999	0.7104	345.10	42	8243.47	4.999	0.6615	344.07	39	8421.86
	0.9	4.999	0.7103	345.16	42	8237.59	4.999	0.6614	344.13	39	8416.01
	1.1	4.999	0.7102	345.27	42	8225.85	4.999	0.6613	344.24	39	8404.32
	1.2	4.999	0.7102	345.32	42	8219.99	4.999	0.6612	344.29	39	8398.48
h	0.56	4.999	0.7107	345.19	42	8234.53	4.999	0.6618	344.16	39	8412.77
	0.63	4.999	0.7105	345.20	42	8233.12	4.999	0.6616	344.17	39	8411.47
	0.77	4.999	0.7101	345.22	42	8230.32	4.999	0.6612	344.19	39	8408.86
	0.84	4.999	0.7099	345.23	42	8228.92	4.999	0.6609	344.20	39	8407.56
C_0	120	4.999	0.7334	328.46	47	10082.9	4.999	0.6836	327.57	44	10252.2
	135	4.999	0.7205	336.83	44	9134.64	4.999	0.6711	335.87	41	9309.00
	165	4.999	0.7025	353.61	39	7373.53	4.999	0.6538	352.50	36	7555.16
	180	4.999	0.6967	362.02	37	6559.62	4.999	0.6481	360.84	34	7533.57

It observed from Table 5.4, the higher value of scale demand α and the constant coefficient of emission reduction function β have a positive impact on total profit, selling price, and green technology investment and slightly reduce cycle time, but the higher value of price elasticity γ has a negative effect on total profit, selling price, and ordering quantity.

The impact of variation of ω_1 is positive proportional to total profit, green investment cost, selling price and ordering quantities but impact of variation of ω_2 is negative proportional to total profit, green investment cost, selling price and ordering quantities. Replenishment cycle time will be reduce for higher value of ω_1 but replenishment cycle time will be enlarge for higher value of ω_2 .

Notice that, if the increase the value e_r and e_h then decrease the total profit. If the increases e_r and e_h , optimal selling price increases and cycle time decreases. It is also noted that an increase in m , selling price and profit increases but an increase in d , selling price and profit decreases. Cycle time and ordering quantity will be increases for the increase the value of m and d .

The percentage rate of interest charge I_c and interest earned I_e directly affect on profit. The higher percentage of I_c reduces the total profit with slightly reduce in cycle time but higher percentage of I_e increases the profit with slightly reduces in cycle time and selling price. The ordering quantity and green investment cost almost remain unchanged for the variation in I_c and I_e .

From Table 5.6, notice that the higher value of inventory cost parameters A, C_0, C_T, h which negative impact on total profit, but selling price will be decreases with increase value of A, C_0, C_T, h and cycle length will be increases due to increasing the value of A but negative effect on cycle length with the higher value of C_0, C_T, h . The order quantity slightly decreases due to increases the cost parameters. The results of variations of A, C_0, C_T, h in case-1 and case-2 on optimum results was found similar as case-3.

From Table 5.4, Table 5.6 and Figure 5.13(a), shows the total profit decreases due to increases ρ , but increases in ρ , resulted in cycle time, selling price and ordering quantity increases.

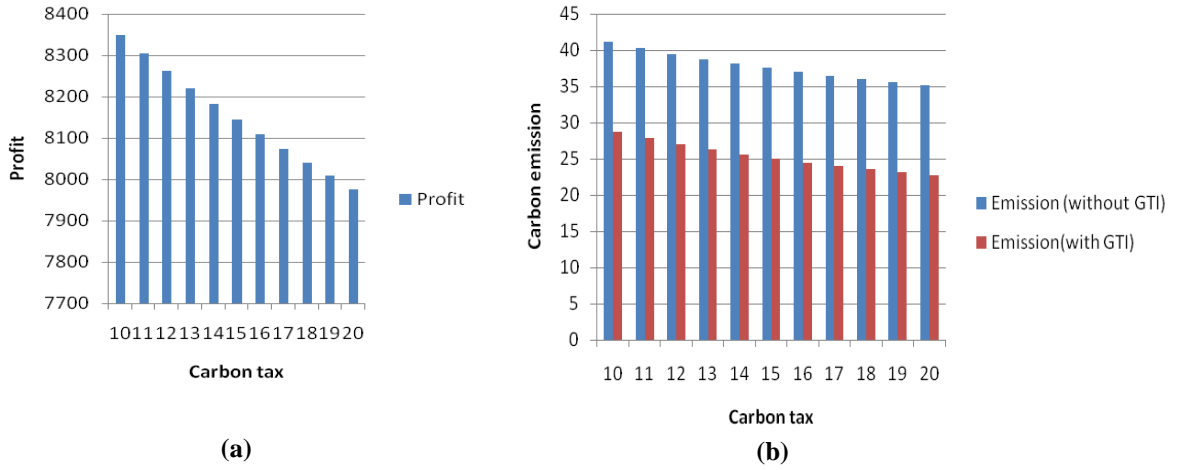


Figure 5.13 Effect of carbon tax on total profit and carbon emission (case-1)

Figure 5.13(b) demonstrates that, if the increases the carbon tax then level of carbon reduction decreases with investing in green technology and without investing in green technology both cases. Here notice from Figure 5.13 and numerical analysis, the carbon reduction is higher, with investing in green investment compare to without investing in green investment.

Table 5.5 and Table 5.6 shows that increases carbon trading price c_{tp} , total profit decreases but cycle time, selling price and order quantity increases. Total profit increases due to increases in c_{cap} but increases in c_{cap} then cycle time, selling price decreases and green investments cost remain unchanged.

The result of variations in parameter α , β , γ , ω_1 , ω_2 , e_T , e_h , m , d , I_c and I_e in case-2 and case-3 on optimum results was found similar as case-1. We can easily notice from Table 5.4 and Table 5.5, sensitivity effect of all parameters on optimum results are same for $M < T$ and $M \geq T$ sub-cases.

Comparative observations of different cases:

From the optimal values of different decision variables, total profit, ordering quantity, and selling price, the following comparison of different cases is made:

Sub-case 1.1 reveals the study about the carbon tax policy and credit period for retailers is lower than cycle time, but sub-case 3.1 indicated only the carbon tax policy, and from the analysis noted that the retailer's profit in sub-case 3.1 is higher than sub-case 3.2. Sub-case

3.1 is more useful compared to sub-case 1.1. Note that the carbon emission units after investing in green technology are lower in sub-case 3.1 compared to sub-case 1.1.

Sub-case 1.2 demonstrates that the carbon tax policy and supplier offer credit period to the retailer is higher than the cycle time. Sub-case 3.1 gives a study of only carbon tax policy. From the optimal results, it is observed that if the cycle time is lower than the credit period, then the profit is higher compared to only the carbon tax policy.

In **sub-case 2.1**, the carbon cap-trade policy and credit period offered by the supplier to the retailer are lower than cycle time, but sub-case 3.1 shows only the cap-trade policy, and as per the study, the retailer's profit in sub-case 2.1 is higher than that in sub-case 3.2.

Sub-case 2.2 shows the carbon cap-trade policy with supplier offer credit period to the retailer is longer than the cycle time. In sub-case 3.2, carbon cap-and-trade is discussed. Here, it is observed that if the credit period is longer than the cycle time, then the retailer gains more profit in sub-case 2.2 compared to sub-case 3.2. As per the analysis, the optimal value of the selling price is lower in the case of trade credit financing, and total profit is also higher if we adopt trade credit financing compared to not adopting trade credit financing for the carbon cap-and-trade policy.

In the comparisons of cases 1 and 2, it is observed that the profit is higher in case 2. If the credit period is longer than the cycle time, the total profit is also higher. In case 2, the carbon emission units after investing in green technology are higher, but the selling price and cycle time are lower, so retailers earn more profit with the extra carbon cap and the trade finance system compared to cases 1 and 3.

5.6 Discussion about managerial insights

From the mathematical formulation and sensitivity analysis of parameters, the following are the managerial insights derived:

- The optimum values of cycle time, green investment cost, and selling price are helpful to retailers in deciding when to replenish orders, how much quantity to replenish, how much to invest in green technology, and what the selling price will be. As a result, the total profit is maximized.(Table 5.2)
- A carbon tax and carbon cap-and-trade mechanism increases a company's desire to minimise carbon emissions, our study indicated that applying carbon policies helpful to decreases level of carbon emissions (Figure 5.13(b)).

- The investment in green technology increases the sustainability of environment by the reducing the carbon emission. Our analysis shown that the carbon emission is lower, if investing in green technology (Figure 5.13(b)).
- To increase profits, retailers should reduce inventory costs. It is evident from the preceding study that the higher value of the ordering cost, purchasing cost, and holding cost, may decrease the total profit. As a result, a retailer must keep the lower rate of inventory cost parameters (Table 5.6).
- The age of perishable products is more important from a business point of view. The product whose shelf life is longer may, consequently, be useful to acquire more profit because the retailer is getting more time to sell the product. Hence, the retailer (manager) should choose the product whose shelf life is longer (Table 5.4).
- The distance between the suppliers and the retailer's warehouse is a prime factor in the retailer's profit. More distance may reduce a retailer's profit because the consumption of fuel and maintenance of vehicles, carbon emissions, delivery time, and other expenses increase. Hence, the minimum distance from the supplier to the retailer's warehouse is beneficial for the retailer (Table 5.4).
- A higher value of the permissible delay period offered by the supplier to the retailer is more beneficial to the retailer from a profit point of view. The trade-credit period is longer than cycle time; retailers earn more profit during the planning horizon because they do not need to pay any interest charges (Table 5.5).
- Retailers or decision-makers should adopt a carbon cap-and-trade regulation with a permissible delay payment policy because a higher carbon cap increases profit while decreasing cycle time and availability of carbon cap is a powerful tool for a cleaner climate (Table 5.2).

5.7 Conclusion

In this chapter, we extended the work of Toptal et al.[23] and Hasan et al.[202] by considering that the product's deterioration is time-dependent and the deterioration rate links to the expiration date, and we presented a new EOQ model with a green investment-sensitive carbon reduction function and a selling price-dependent demand and trade credit payment policy. A mathematical formulation of the model is an attempt to formulate an inventory system in order to maximize the retailer's total profit with respect to the optimal selling price, replenishment time, and green technology investment cost using classical

optimization. The suggested model has been verified through the numerical examples, and the optimal strategies have been authenticated by doing a sensitivity analysis on the optimal solutions. Our analysis shows six different sub-cases to find out the best inventory strategy. Some of the key findings derived from the chapter are: (i) carbon cap-and-trade policy with trade credit financing is better than carbon tax policy; (ii) optimization of selling prices and green investment strategies are subject to carbon reduction with carbon policies; (iii) efficiency factor of green technology directly affects optimal solutions and increases profit; (iii) emission factor of green technology negatively affects profit; (iv) since the carbon tax has a substantial impact on the overall profit, the authorities properly regulate the tax rate; (v) longer expiration dates for products as well as a proper credit period have a positive impact on optimal decisions.

Future investigations may expand on this research in a number of different ways. It can be extended by incorporating preservation technology for reducing deterioration, advanced payment policies, markdown pricing policies, a developed production quantity model. Additionally, the demand sensitive to products freshness and selling price is also a better extension. The study is only concerned with the deterioration of perishable goods. Both perishable and non-perishable products may be taken into account by future investigators in order to strengthen their studies.

CHAPTER-6

Sustainable Economic Production Quantity (SEPQ) Model for Inventory having Green Technology Investments - Price Sensitive Demand with Expiration Dates

6.1 Introduction

The practice of incorporating various environmental concerns into production-inventory models has grown significantly. The importance of reducing carbon emissions from manufacturing and commercial activity has grown in the past few decades. Due in significant part to carbon laws implemented by various governing bodies, industries are currently seeking ways to mitigate the amount of greenhouse gases linked to the operations they conduct. Investment in green technology is one of the best ways industries can reduce carbon emissions due to the production process and their operations and is helpful to environmental sustainability. The demand is depends on environmental sustainability Zaroni et al.[197]. A large number of analysts endorse a carbon tax and cap as a strategy for cutting carbon emissions. In inventory models, one of the factors is the deterioration rate, which is usually assumed to be constant; however, this may not always be possible for each commodity. It has been observed that perishable products decay gradually at first, then rapidly as their expiration date approaches and other factors such as product selling price play a major role in the inventory system. In light of this, we developed a manufacturer's sustainable economic production quantity inventory model (SEPQ) with

green technology investment and selling price-sensitive demand in this chapter. Furthermore, products have a time-varying deterioration rate, which also depends on the expiration date of the products. Setting up the production system, manufacturing process, holding inventory, deterioration of products, and environmental impact are sources of carbon emissions considered in the proposed chapter. Carbon tax and cap policy, and green technology investment is implemented to achieve sustainability. The main objective of the study is to find the optimal replenishment time, optimal green investment, and optimal selling price by considering manufacturer's profit maximization using classical optimization. To support the validity of the sustainable economic production quantity model, a numerical example has been explored. The management implications of the best feasible solution with respect to parameters have been revealed via sensitivity analysis. A few concluding comments and potential future applications are then presented.

6.2 Notations and Assumptions

The below notations and assumptions are implemented to build the proposed chapter.

6.2.1 Notations

Parameters

A	Retailer ordering cost (in ₹/order)
P	Constant production rate (unit/year)
k	Production cost per cycle (in ₹/unit)
ρ	Carbon tax per cycle (in ₹/kilogram)
c_{cap}	Carbon emissions cap (kilogram /year)
Q	Maximum inventory level when production stops at $t = T_1$
T_1	Point of time at which production stops (year)
m	Maximum shelf life of products in units of time, i.e. expiration date and product can not be sold after m , $m > 0$.
ξ	fraction of carbon emissions after green technology investment $0 < \xi < 1$
η	Efficiency of greener technology in reducing emission $\eta > 0$.

b	Space required for each unit of product (meters/unit)
e_h	Carbon emission for inventory holding per cycle (kilogram/unit /year)
e_{sp}	Carbon emissions unit associated in setup cost (kilogram/unit /year)
e_m	Carbon emission from manufacturing process (kilogram/unit /year)
e_{ei}	Environmental impact carbon emissions for inventory (kilogram/unit /year)
e_{dp}	Emission per deteriorated products (kilogram /unit/year)

Decision variables

g	Green technology investment cost (in ₹/ unit/cycle) (a decision variable)
p	Selling Price (in ₹/unit) (a decision variable)
T	Length of inventory cycle (years) (decision variable).

Expressions and functions

$f(g)$	The fraction of carbon reduction.
$R(g, p)$	Demand function depending on green investment cost and selling price.
$\theta(t)$	Deterioration rate as a time-varying function.
$I_1(t)$	Inventory level during $0 \leq t \leq T_1$ (units).
$I_2(t)$	Inventory level during $T_1 \leq t \leq T$ (units).

Objective function

$TP(g, T, p)$	Manufacturer total profit function
---------------	------------------------------------

6.2.2 Assumptions

1. The manufacturer produced a single type of product.
2. Lead time to be considered negligible. Replenishment rate is instantaneous.
3. Rate of production is constant and more than a demand rate, shortages are avoided.
A product unit must be sold after it has been produced.
4. Investment in green technologies should be taken into consideration to decrease the impact of carbon emissions. The fraction of reduction of average emission is $f(g) = \xi(1 - e^{-\eta g})$; where, $0 < \xi < 1$ is the fraction of carbon emission after investing in green technology, $\eta > 0$ efficiency of greener technology in reducing emission and $g > 0$ is the green investment cost. The expression

$$f(g) = \xi(1 - e^{-\eta g}) \Rightarrow g = -\frac{1}{\eta} \left(\ln(1 - \frac{f}{\xi}) \right) \text{ gives, if } g \rightarrow 0 \text{ then } f \rightarrow 0 \text{ and } g \rightarrow \infty$$

then $f \rightarrow \xi$ and $f(g)$ is continuous differentiable function with $f'(g) > 0$, $f''(g) < 0$. (Lou et al.[174], Mishra et al.[24])

5. The manufacturer invests in green technology for sustainability over a certain time frame without raising the unit price of the product.
6. The reduction in carbon emissions by green investments and the selling price of products are both directly affected by market demand. The sustainability affect positive on buyer's decisions and investment in green technology impact positively on environment and buyer's demand ,Zanoni et al.[197], Hasan et. al.[202]. Demand function is a linear form of carbon reduction function and selling price, It can be defined as $R(g, p) = \alpha + \beta f(g) - \gamma p$, $0 \leq t \leq T$, where $\alpha > 0$ is the scale demand, $\beta > 0$ is the constant coefficient of $f(g)$ and $0 < \gamma < 1$ is the price elasticity.
7. Product deteriorate continuously with time, product cannot be sold after expiration date m , and we assumed that the rate deterioration $\theta(t) = \frac{1}{1+m-t}$, $0 \leq t \leq T \leq m$, if $t \rightarrow 0$ then deterioration rate is minimum, and $t \rightarrow m$ then all products deteriorate as its expiration date.
8. The replenishment cycle time is shorter than the maximum feasible product life span m . i.e. $T \leq m$.
9. Replacement, repair, salvage value of deteriorate products is avoided.
10. The carbon footprint of the setup production system, manufacturing process, inventory holding operations, inventory deterioration, and environmental impact are all taken into account (Mishra et. al[24]).
11. Carbon tax and cap strategies applied for managing carbon emission.

6.3 Mathematical formulation

In this section, a sustainable EPQ model is developed in which deterioration of products is time dependent. The manufacturing process start at time $t = 0$ and goes up to time $t = T_1$, at T_1 the inventory level goes to its highest level. Production process stops at time T_1 , and

inventory level goes down due to the demand and deterioration. Inventory level after $t = T_1$ goes down to zero at $t = T$. It is observed that the rate of inventory level increases due to the production rate and decreases due to demand and deterioration rate. The following differential equation formulates the changing of inventory level.

$$\frac{dI_1(t)}{dt} + \theta(t)I_1(t) = -(R(g, p) - P), 0 \leq t \leq T_1 \quad (6.1)$$

with initial condition $I_1(0) = 0$.

Now, duration the period $[T_1, T]$, noticed that the inventory level consumed due to demand of item and deterioration effect on produced item, so the governing differential equation in this non production period is given by;

$$\frac{dI_2(t)}{dt} + \theta(t)I_2(t) = -R(g, p), T_1 \leq t \leq T \quad (6.2)$$

With the end inventory level $I_2(T) = 0$. The solution of the differential equations (6.1) and (6.2) with given conditions is respectively,

$$I_1(t) = (R(g, p) - P)(1 + m - t) \ln \left(\frac{1 + m - t}{1 + m} \right), 0 \leq t \leq T_1, \quad (6.3)$$

$$I_2(t) = (R(g, p))(1 + m - t) \ln \left(\frac{1 + m - t}{1 + m - T} \right), T_1 \leq t \leq T \quad (6.4)$$

The inventory functions are continues at $t = T_1$, i.e. $I_1(T_1) = I_2(T_1)$, the relation between T_1 and T can be derived as,

$$T_1 = (1 + m) \left(1 - \left(\frac{1 + m - T}{1 + m} \right)^{\frac{R(g, p)}{P}} \right) \quad (6.5)$$

Using the boundary condition $I_1(T_1) = Q$, the maximum produces products are

$$Q = (R(g, p) - P) \left((1 + m) \left(\frac{1 + m - T}{1 + m} \right)^{\frac{R(g, p)}{P}} \right) \ln \left(\frac{\left((1 + m) \left(\frac{1 + m - T}{1 + m} \right)^{\frac{R(g, p)}{P}} \right)}{1 + m} \right) \quad (6.6)$$

Our objective is to maximize the manufacturer's total profit. The total annual profit consists of the following components.

Net sales revenue from the selling the product:

$$SR = \frac{p}{T} \left(\int_0^T R(g, p) dt \right) = pR(g, p) \quad (6.7)$$

The cost of production is over the cycle:

$$PDC = \frac{k}{T_1} \left(\int_0^{T_1} R(g, p) dt \right) = kR(g, p) \quad (6.8)$$

The annual fixed setup cost has calculated as:

$$STC = \frac{A}{T} \quad (6.9)$$

The holding cost per cycle:

$$HC = \frac{h}{T} \left[\int_0^{T_1} I_1(t) dt + \int_{T_1}^T I_2(t) dt \right] \quad (6.10)$$

Green technology investment cost per year:

$$GTC = \frac{gT}{T} = g \quad (6.11)$$

The emission associated in setup production:

$$E_{sp} = \frac{e_{sp}}{T} \quad (6.12)$$

The emission from holding process of inventory:

$$E_h = \frac{e_h b}{T} \left[\int_0^{T_1} I_1(t) dt + \int_{T_1}^T I_2(t) dt \right] \quad (6.13)$$

The emission from manufacturing process:

$$E_m = \frac{e_m}{T_1} \left(\int_0^{T_1} R(g, p) dt \right) = e_m R(g, p) \quad (6.14)$$

The emissions due to environmental impact:

$$E_{ei} = \frac{e_{ei}}{T} \left(\int_0^T R(g, p) dt \right) = e_{ei} R(g, p) \quad (6.15)$$

The number of deteriorated products during the cycle time:

$$DI = \frac{1}{T} \left(Q - \left(\int_0^{T_1} R(g, p) dt + \int_{T_1}^T R(g, p) dt \right) \right)$$

Emission due to deteriorating of product during the cycle:

$$E_{dp} = \frac{e_{dp}}{T} \left(Q - \left(\int_0^{T_1} R(g, p) dt + \int_{T_1}^T R(g, p) dt \right) \right) \quad (6.16)$$

Total carbon emission from (6.12) to (6.16) is,

$$\hat{C} = E_{sp} + E_h + E_m + E_{ei} + E_{dp} \quad (6.17)$$

The fraction of carbon reduction function $f(g) = \xi(1 - e^{-\eta g})$ is taken as per Lou et al.[174], the total carbon emission after applying green technology investment is,

$$\hat{C}_g = \hat{C}(1 - \xi(1 - e^{-\eta g})) \quad (6.18)$$

Hence, the annual profit of the manufacturer is under a carbon cap and tax functions as below:

$$\begin{aligned} TP(g, p, T) &= (SR - PDC - STC - HC - GTC) + \rho[c_{cap} - \hat{C}(1 - \xi(1 - e^{-\eta g}))] \\ &= \left[pR(g, p) - kR(g, p) - \frac{A}{T} - \frac{h}{T} \left[\int_0^{T_1} I_1(t) dt + \int_{T_1}^T I_2(t) dt \right] - g \right] \\ &\quad + \rho[c_{cap} - \left(\frac{e_{sp}}{T} + \frac{e_h b}{T} \left[\int_0^{T_1} I_1(t) dt + \int_{T_1}^T I_2(t) dt \right] + e_m R(g, p) + e_{ei} R(g, p) \right) \\ &\quad + \frac{e_{dp}}{T} \left(Q - \left(\int_0^{T_1} R(g, p) dt + \int_{T_1}^T R(g, p) dt \right) \right)] (1 - \xi(1 - e^{-\eta g})) \end{aligned} \quad (6.19)$$

Here notice that the manufacturer total carbon emission cost is $\rho[c_{cap} - \hat{C}_g]$ included green technology investment and carbon cap-tax policy. The value of $\rho[c_{cap} - \hat{C}_g]$ is positive then manufacturer should sell the remaining carbon quota and earn extra revenue. If increased the total carbon emission from his/her total carbon quota then the manufacturer must buy a extra carbon quota.

6.3.1 Solution technique to determine the optimal solution and concavity:

To find optimal value of g , p and T and to prove the concavity of equation (6.19) with respected to g , p and T , the following methodology to be adopted.

Theorem 6.1: *For any given distinct value of T and fixed positive value of p , then*

(a) *The equation $\frac{\partial TP}{\partial g}$ has one and only one solution.*

(b) *The sufficient conditions for maxima satisfied by the value obtained in (a).*

Proof: Any discrete value of T and fixed positive value of p , the first and second order partial derivative of (6.19) with respect to g ; the following results can be found:

$$\begin{aligned} \frac{\partial TP}{\partial g} = & \left[p\beta\xi(\eta e^{-\eta g}) - k\beta\xi\eta e^{-\eta g} - 0 - h \frac{\partial}{\partial g} \left[\frac{X}{T} \right] - 1 \right] \\ & + \left[\rho c_{cap}(-\eta e^{-\eta g}) - \left[\begin{aligned} & \frac{e_{sp}}{T} \xi(-\eta e^{-\eta g}) + e_h b \frac{\partial}{\partial g} \left[(1 - \xi(1 - e^{-\eta g})) \left[\frac{X}{T} \right] \right] + \\ & (e_m + e_{ei}) \frac{\partial}{\partial g} \left[(1 - \xi(1 - e^{-\eta g})) (\alpha + \beta\xi(1 - e^{-\eta g}) - \gamma p) \right] \\ & + e_{dp} \frac{\partial}{\partial g} \left[(1 - \xi(1 - e^{-\eta g})) \left[\frac{Y}{T} \right] \right] \end{aligned} \right] \right] = 0 \quad (6.20) \end{aligned}$$

where,

$$X = \left[\begin{aligned} & \int_0^{T_1} \left\{ -(P - R(g, p))(1 + m - t) \ln \left(\frac{1 + m - t}{1 + m} \right) \right\} dt \\ & + \int_{T_1}^T \left\{ (R(g, p))(1 + m - t) \ln \left(\frac{1 + m - t}{1 + m - T} \right) \right\} dt \end{aligned} \right] \quad (6.21)$$

$$\text{and } Y = Q - \left(\int_0^{T_1} R(g, p) dt + \int_{T_1}^T R(g, p) dt \right) \quad (6.22)$$

$$\begin{aligned}
\frac{\partial^2 TP}{\partial g^2} = & \left[p\beta\xi(-\eta^2 e^{-\eta g}) + k\beta\xi\eta^2 e^{-\eta g} - h \frac{\partial^2}{\partial g^2} \left[\frac{X}{T} \right] \right] \\
& + \left[\begin{aligned} & \left[\frac{e_{sp}}{T} \xi(\eta^2 e^{-\eta g}) + e_h b \frac{\partial^2}{\partial g^2} \left[(1 - \xi(1 - e^{-\eta g})) \left[\frac{X}{T} \right] \right] + \right. \\ & \left. \rho c_{cap}(\eta^2 e^{-\eta g}) - (e_m + e_{ei}) \frac{\partial^2}{\partial g^2} \left[(1 - \xi(1 - e^{-\eta g}))(\alpha + \beta\xi(1 - e^{-\eta g}) - \gamma p) \right] \right. \\ & \left. + e_{dp} \frac{\partial^2}{\partial g^2} \left[(1 - \xi(1 - e^{-\eta g})) \left[\frac{Y}{T} \right] \right] \right] \end{aligned} \right] \quad (6.23)
\end{aligned}$$

$$\text{But, } p > k, \frac{\partial^2}{\partial g^2} \left[\frac{X}{T} \right] > 0, \frac{\partial^2}{\partial g^2} \left[(1 - \xi(1 - e^{-\eta g})) \left[\frac{X}{T} \right] \right] > 0, \frac{\partial^2}{\partial g^2} \left[(1 - \xi(1 - e^{-\eta g})) \left[\frac{Y}{T} \right] \right] > 0$$

The conditions indicated that the value of (6.23) is less than zero.

Hence $\frac{\partial^2 TP}{\partial g^2} < 0$, so the g^* is a unique value of (6.20) and profit function is concave at g^* .

Theorem 6.2: For any discrete positive value of T and fixed positive value of g , then

(a) The equation $\frac{\partial TP}{\partial p} = 0$ has one and only one solution.

(b) The sufficient condition for maxima satisfied by the value obtained in (a).

Proof: For any discrete value of T and fixed positive value of g , taking the first partial derivatives of Eq. (6.19) with respect to p ; the following results can be found:

$$\begin{aligned}
\frac{\partial TP}{\partial p} = & \left[\alpha + \beta\xi(1 - e^{-\eta g}) - 2\gamma p + k\gamma - h \frac{\partial}{\partial p} \left[\frac{X}{T} \right] \right] \\
& + \left[\begin{aligned} & \left(\rho e_h b \frac{\partial}{\partial p} \left[\frac{X}{T} \right] - \rho\gamma(e_m + e_{ei}) \right) \\ & + \rho e_{dp} \frac{\partial}{\partial p} \left[\frac{Y}{T} \right] \end{aligned} \right] (1 - \xi(1 - e^{-\eta g})) = 0 \quad (6.24)
\end{aligned}$$

In (6.24), the value of X and Y is as per the (6.21) and (6.22) respectively.

Taking the second order partial derivative of (6.19) with respect to p , we have

$$\frac{\partial^2 TP}{\partial p^2} = \left[-2\gamma - h \frac{\partial^2}{\partial p^2} \left[\frac{X}{T} \right] \right] + \left[- \left(\rho e_h b \frac{\partial^2}{\partial p^2} \left[\frac{X}{T} \right] + \rho e_{dp} \frac{\partial^2}{\partial p^2} \left[\frac{Y}{T} \right] \right) (1 - \xi(1 - e^{-\eta g})) \right] \quad (6.25)$$

But $\frac{\partial^2}{\partial p^2} \left[\frac{X}{T} \right] > 0$, $\frac{\partial^2}{\partial p^2} \left[\frac{Y}{T} \right] > 0$ and $0 < 1 - \xi(1 - e^{-\eta g}) < 1$, it indicated that the total value of

(6.25) is lower than zero. We concluded that $\frac{\partial^2 TP}{\partial p^2} < 0$.

It's shown that selling price have a unique value exist and second order sufficient condition satisfied at the optimal value of . so, TP is concave at p^* .

Theorem 6.3: For any positive value of p and fixed positive value of g , then

(a) The equation $\frac{\partial TP}{\partial T} = 0$ has one and only one solution.

(b) The sufficient condition for maxima satisfied by the value obtained in (a).

Proof: Take the first order partial derivative of profit function (6.19) with respect to T for any positive value of p and fixed positive value of g , we have

$$\frac{\partial TP}{\partial T} = \left[\frac{A}{T^2} - h \frac{\partial}{\partial T} \left[\frac{X}{T} \right] \right] + \left[\left(-\frac{\rho e_{sp}}{T^2} + \rho e_h b \frac{\partial}{\partial T} \left[\frac{X}{T} \right] \right) (1 - \xi(1 - e^{-\eta g})) \right. \\ \left. + \rho e_{dp} \frac{\partial}{\partial T} \left[\frac{Y}{T} \right] \right] = 0 \quad (6.27)$$

In (6.27), the value of X and Y is as per the (6.21) and (6.22) respectively,

Again taking the partial derivative of (6.27) with respect to T , we get the expression as,

$$\frac{\partial^2 TP}{\partial T^2} = \left[-\frac{2A}{T^3} - h \frac{\partial^2}{\partial T^2} \left[\frac{X}{T} \right] \right] + \left[-\left(\frac{2\rho e_{sp}}{T^3} + \rho e_h b \frac{\partial^2}{\partial T^2} \left[\frac{X}{T} \right] \right) (1 - \xi(1 - e^{-\eta g})) \right. \\ \left. + \rho e_{dp} \frac{\partial^2}{\partial T^2} \left[\frac{Y}{T} \right] \right] \quad (6.28)$$

We can observe from (6.28) for the any positive value of p and g , $\frac{\partial^2}{\partial T^2} \left[\frac{X}{T} \right] > 0$,

$\frac{\partial^2}{\partial T^2} \left[\frac{Y}{T} \right] > 0$ and $0 < 1 - \xi(1 - e^{-\eta g}) < 1$. Hence, $\frac{\partial^2 TP}{\partial T^2} < 0$.

The decision parameter T have a unique optimal value at profit function concave at the optimal value of T .

Theorem 6.4: *For any fix positive value of g , the total profit of manufacturer $TP(g, p, T)$ expressed in (6.19) is maximum value if determinant of Hessian matrix is greater than zero.*

Proof : First we take the partial derivative of (6.19) with respect to p and T , we have

$$\frac{\partial^2 TP}{\partial p \partial T} = [hX_1] + \left[(\rho e_h b X_1 + \rho e_{dp} Y_1) (1 - \xi(1 - e^{-\eta g})) \right] \quad (6.29)$$

where $X_1 = \frac{\partial^2}{\partial p \partial T} \left[\frac{X}{T} \right]$ and $Y_1 = \frac{\partial^2}{\partial p \partial T} \left[\frac{Y}{T} \right]$,

Consider the hessian matrix,

$$H = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} = \begin{bmatrix} \frac{\partial^2 TP(g, p, T)}{\partial p^2} & \frac{\partial^2 TP(g, p, T)}{\partial p \partial T} \\ \frac{\partial^2 TP(g, p, T)}{\partial T \partial p} & \frac{\partial^2 TP(g, p, T)}{\partial T^2} \end{bmatrix}$$

The determinant of hessian matrix should be calculated by numerically and checked positive or not when g considering fixed value.

From the (6.25) and (6.28), it is noticed that $H_{11} < 0$ and $H_{22} < 0$. The determinant of the Hessian matrix is for the positive value of g .

$$\frac{\partial^2 TP(g, p, T)}{\partial p^2} \frac{\partial^2 TP(g, p, T)}{\partial T^2} > \left(\frac{\partial^2 TP(g, p, T)}{\partial p \partial T} \right)^2 \Rightarrow \det(H) > 0 \quad (6.30)$$

Hence the total profit is maximum and unique exist. The total profit is concave with respect to all decision variable has shown in next numerical example section.

Solution Algorithm:

Step 1: Use the mathematical software like Maple18 or Matlab or Mathematica and taking initial parameters mentioned above in (6.19).

Step 2: Set $g = 0$

Step 3: Evaluate p from (6.24), T from (6.27).

Step 4: Check sufficient conditions $H_{11} < 0$, $H_{22} < 0$ and $H_{11}H_{22} - H_{12}H_{21} > 0$. Otherwise choose different parametric value in step 1.

Step 5: Increase the value of g from 0 and repeat Step 3 until to get maximum value of $TP(g, p, T)$.

Step 6: Evaluate total carbon emission from (6.18) and production quantity from (6.6).

Step 7: Obtain manufacturer's total profit using (6.19).

Step 8: Stop.

Next, let's use the numerical example below to show the described model so that you may grasp it better.

6.4 Numerical example

We consider following example to validate the mathematical formulation.

Example 6.4.1: The numerical values of the parameters in correct units were used as input for the model's numerical, graphical, and sensitivity analyses. The unit of parameters are mentioned in the notation section 6.2.1.

$$\alpha = 150, \beta = 10, \gamma = 0.7, k = 6, e_h = 4, h = 6, c_{cap} = 900, e_m = 40, e_{ei} = 60, \\ \rho = 0.33, P = 700, A = 80, e_{dp} = 30, e_{sp} = 60, b = 0.4, \xi = 0.2, \eta = 0.8, m = 1$$

As per the solution procedure and algorithm described in previous section, the optimal value of decision variables, total profit, and production quantity and carbon emission units derived as below:

Table 6.1 Output value as per solution algorithm

g^*	T^*	p^*	\hat{C}_g	TP^*	Q^*
0	0.3881456182	125.3513237	2099.67636	5351.19940	24
5	0.4013775110	123.7673693	1763.46649	5946.66697	25
7.77	0.4016376947	123.7416265	1757.56279	5954.10798	26
9	0.4016576952	123.7396542	1757.10994	5953.66084	26
12	0.4016685541	123.7385838	1756.86405	5951.08582	27

***bold value specified the optimal results;** From the numerical experiment the sufficient conditions $H_{11} = -1.35 < 0$, $H_{22} = -3318 < 0$ and $H_{11}H_{22} - H_{12}H_{21} = 4474 > 0$ are satisfied.

Optimum value of green investment cost is $g^* = ₹7.7702$, manufacture's selling price $p^* = ₹123.7416$ and production cycle time is $T^* = 0.4016$ year.

The maximum profit of manufacturer's is **₹5954.11**. The optimum production quantities produced by manufacturer are $Q^* = 26$ **units**. Total carbon emission after investing in green

technology is **1757.56** kg/year/unit and without investing in green technology the carbon emission is **2099.68** kg/year/unit.

6.4.1 Graphical authentication of the concavity of objective function

Using the numerical values of parameters as mentioned above, the concavity behaviour of profit function is shown in Figure 6.1, Figure 6.2 and Figure 6.3 with respect to decision variables.

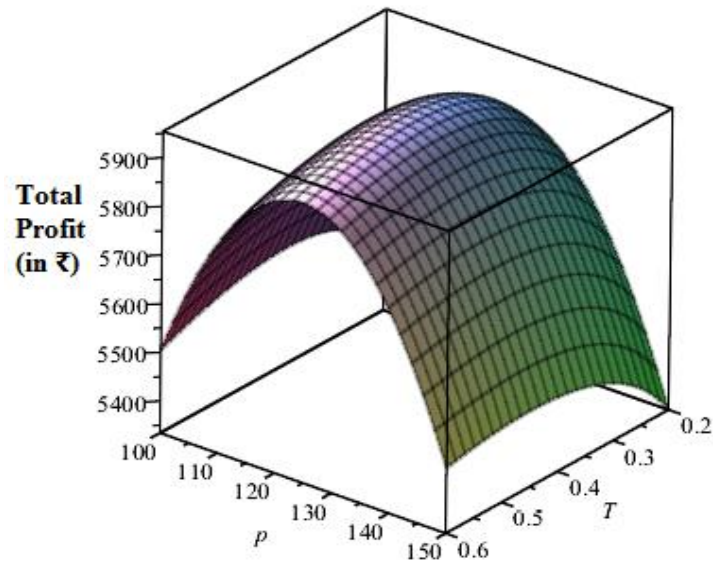


Figure 6.1 Concavity of the objective function with respect to p and T

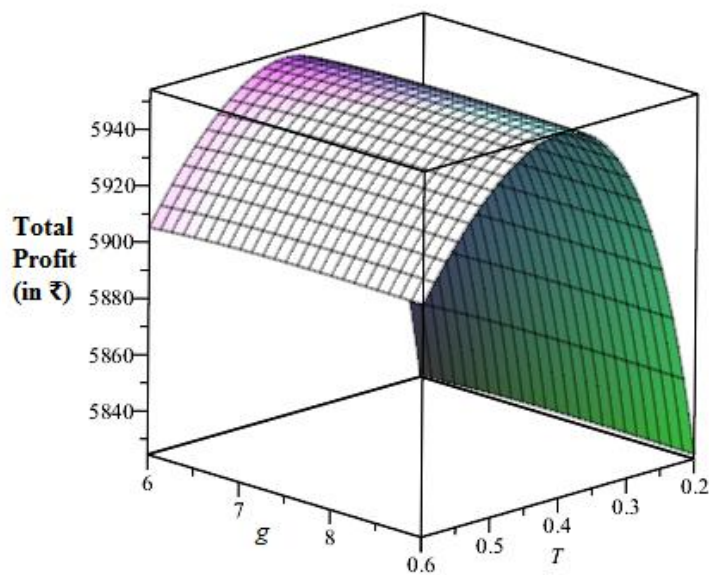


Figure 6.2 Concavity of the objective function with respect to g and T

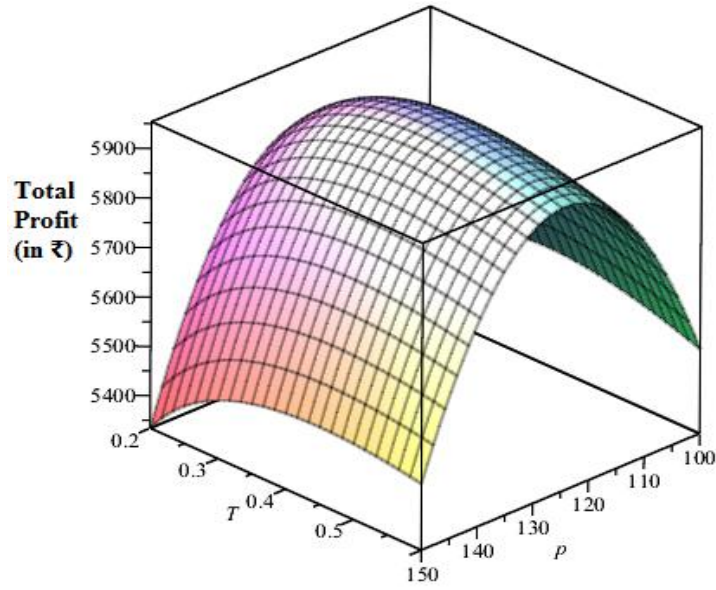


Figure 6.3 Concavity of the objective function with respect to T and p

6.5 Sensitivity analysis and observations

Based on the numerical example used in Section 6.4, we now examine the impact of changing the system's parameters on the optimal values. Sensitivity analysis is carried out by altering each parameter's value in turn by -20%, -10%, +10%, and +20% while leaving the other parameter's value unaffected.

Table 6.2 Sensitivity analysis of system parameters

Parameters	Values	g^* (in ₹)	T^* (Year)	p^* (in ₹)	Q^* (Unit)	\hat{C}_g	TP^* (in ₹)
α	120	7.46	0.4400	103.19	23	1391.84	3486.59
	135	7.63	0.4187	113.45	24	1577.67	4636.69
	165	7.90	0.3877	134.05	28	1931.86	7438.44
	180	8.01	0.3762	144.38	29	2100.81	8972.25
β	8	7.69	0.4020	123.47	26	1752.90	5916.89
	9	7.73	0.4018	123.60	26	1755.23	5935.48
	11	7.81	0.4014	123.88	26	1759.90	5972.77
	12	7.85	0.4012	124.02	26	1762.23	5991.45
γ	0.56	7.90	0.3974	150.77	26	1807.06	7984.88
	6.3	7.83	0.3995	135.75	26	1782.46	6854.83
	0.77	7.72	0.4039	113.93	26	1732.36	5220.20
	0.84	7.67	0.4061	105.77	26	1706.83	4611.43
P	560	7.76	0.4137	122.99	26	1724.13	6005.11
	630	7.76	0.4069	123.41	26	1742.86	5976.67
	770	7.78	0.3975	124.01	26	1769.41	5935.76

Parameters	Values	g^* (in ₹)	T^* (Year)	p^* (in ₹)	Q^* (Unit)	\hat{C}_g	TP^* (in ₹)
	840	7.78	0.3942	124.24	26	1779.16	5920.55
A	64	7.77	0.3699	123.64	24	1750.39	5995.57
	72	7.77	0.3862	123.69	25	1753.98	5974.42
	88	7.77	0.4163	123.79	27	1761.13	5934.55
	96	7.77	0.4302	123.84	28	1764.67	5915.65
k	4.8	7.78	0.4008	123.12	26	1767.47	6032.82
	5.4	7.77	0.4012	123.43	26	1762.52	5993.40
	6.6	7.77	0.4021	124.05	26	1752.60	5914.95
	7.2	7.76	0.4025	124.36	26	1747.64	5875.92
h	4.8	7.77	0.4143	123.68	27	1762.43	5969.56
	5.4	7.77	0.4078	123.71	26	1759.93	5961.77
	6.6	7.77	0.3957	123.77	26	1755.31	5946.57
	7.2	7.77	0.3900	123.81	25	1753.18	5939.16
c_{cap}	720	7.77	0.4016	123.74	26	1757.56	5894.71
	810	7.77	0.4016	123.74	26	1757.56	5924.41
	990	7.77	0.4016	123.74	26	1757.56	5983.81
	1080	7.77	0.4016	123.74	26	1757.56	6013.51
ρ	0.264	7.61	0.3928	120.98	26	1435.28	6254.25
	0.297	7.69	0.3972	122.36	26	1599.08	6102.87
	0.363	7.84	0.4061	125.13	26	1910.74	5807.95
	0.396	7.90	0.4106	126.51	26	2058.62	5664.40
ξ	0.16	7.48	0.3985	124.06	26	1829.22	5829.76
	0.18	7.63	0.4001	123.90	26	1793.58	5891.78
	0.22	7.89	0.4033	123.58	26	1721.16	6016.73
	0.24	8.01	0.4049	123.43	26	1684.37	6079.64
η	0.64	9.36	0.4016	123.74	26	1757.74	5952.20
	0.72	8.49	0.4016	123.74	26	1757.64	5953.25
	0.88	7.17	0.4016	123.74	26	1757.50	5954.82
	0.96	6.67	0.4016	123.74	26	1757.44	5955.42
e_h	3.2	7.77	0.4025	123.74	26	1756.82	5955.18
	3.6	7.77	0.4021	123.74	26	1757.19	5954.64
	4.4	7.77	0.4012	123.74	26	1757.93	5953.57
	4.8	7.77	0.4008	123.75	26	1758.31	5953.04
e_m	32	7.71	0.4001	122.64	26	1635.27	6093.07
	36	7.74	0.4009	123.19	26	1696.83	6023.38
	44	7.80	0.4024	124.29	26	1817.47	5885.24
	48	7.82	0.4032	124.84	26	1876.56	5816.78
e_{ei}	48	7.68	0.3994	122.10	26	1572.88	6163.15
	54	7.73	0.4005	122.92	26	1666.15	6058.17
	66	7.81	0.4028	124.56	26	1847.12	5850.96
	72	7.85	0.4039	125.39	26	1934.82	5748.72
e_{sp}	48	7.77	0.3956	123.72	26	1748.13	5962.06
	54	7.77	0.3986	123.73	26	1752.88	5958.07
	66	7.77	0.4046	123.75	26	1762.19	5950.18
	72	7.77	0.4075	123.76	26	1766.75	5946.27
e_{dp}	24	7.77	0.4273	124.14	28	1751.33	5960.43
	27	7.77	0.4139	123.94	27	1754.07	5957.68
	33	7.77	0.3905	123.54	25	1761.71	5949.82
	36	7.77	0.3802	123.33	25	1766.45	5944.90
m	0.8	7.77	0.4016	123.74	26	1774.74	5936.33
	0.9	7.77	0.4016	123.74	26	1765.63	5945.76
	1.1	7.77	0.4016	123.74	26	1750.37	5961.55
	1.2	7.77	0.4016	123.74	26	1743.92	5968.22

- Table 6.2 and Figure 6.4 explored the impact of the parameters on optimal total profit TP^* . The constant demand α and the coefficient of emission reduction function β are a positive proportional to the total profit. Profit will be decreases if increasing the product rate P . It is obvious that the profit will be decreased due to the increases the value of cost parameters A , k and h . The higher value of c_{cap} will be positive effect in profit but the total profit reduces due to higher value of ρ . If the increases the parameters ξ and η then carbon reduction \hat{C}_g is decreased, consequently profit is increases. On other hand, profit will be decreases the impact of emission parameters $e_h, e_m, e_{ei}, e_{sp}, e_{dp}$. It is observed that, if increases m then total profit also increases.
- It is observed that the carbon emission \hat{C}_g increases heavily, when the parameters $\alpha, \beta, P, \eta, e_h, e_m, e_{ei}, e_{sp}$ and e_{dp} increases. If the increases the value of γ, ξ and cost parameters k, h, A increases then carbon emission decreases. Other parameters have a minor effect on carbon emission.

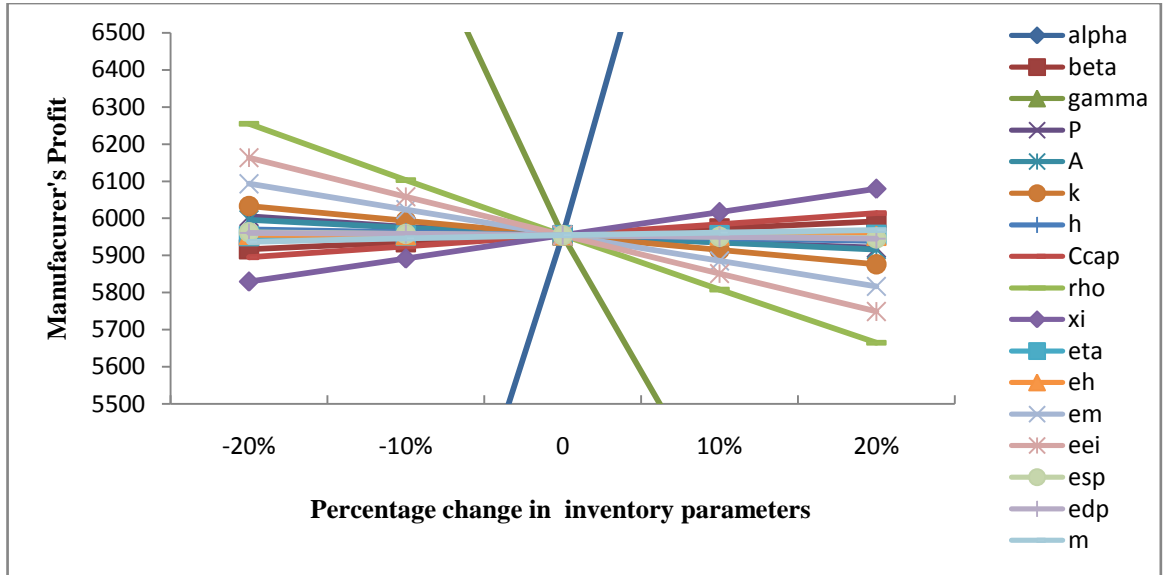


Figure 6.4 Effect of inventory parameters on manufacturer's profit

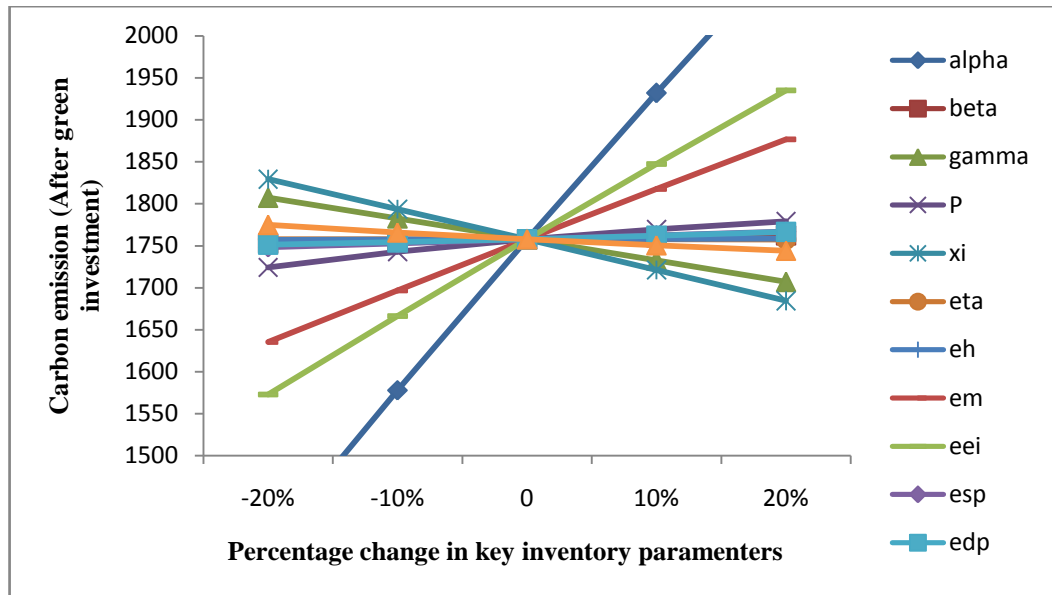


Figure 6.5 Effect of inventory parameters on carbon emission

- Figure 6.6 and Table 6.2 described that green investment cost have a influence due to changes value of parameters. A remarkable effect on green investment cost by changes the parameters $\alpha, \beta, P, \gamma, P, \xi$. If the increases in $\alpha, \beta, P, \rho, \xi, e_m$ and e_{ei} resulted into increase in green investment cost. On the other hand, the green investment cost decreases with increases in γ, k . Minor influence on green investment cost by the parameters A, h, e_{sp}, e_{dp} and m . The parameter η significantly impacted on green investment cost, if the increase in η , highly decreases the green investment cost.

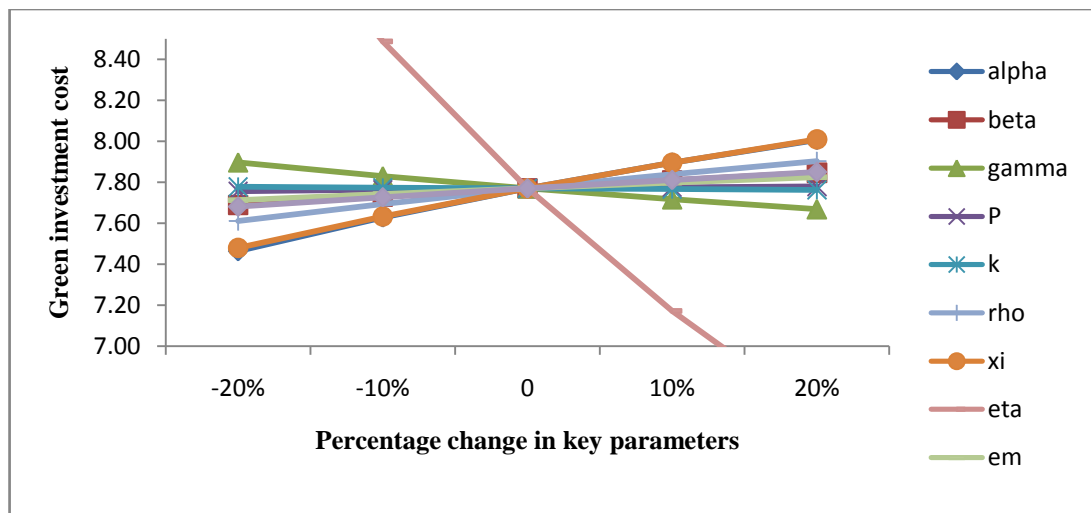


Figure 6.6 Effect of inventory parameters on green investment cost

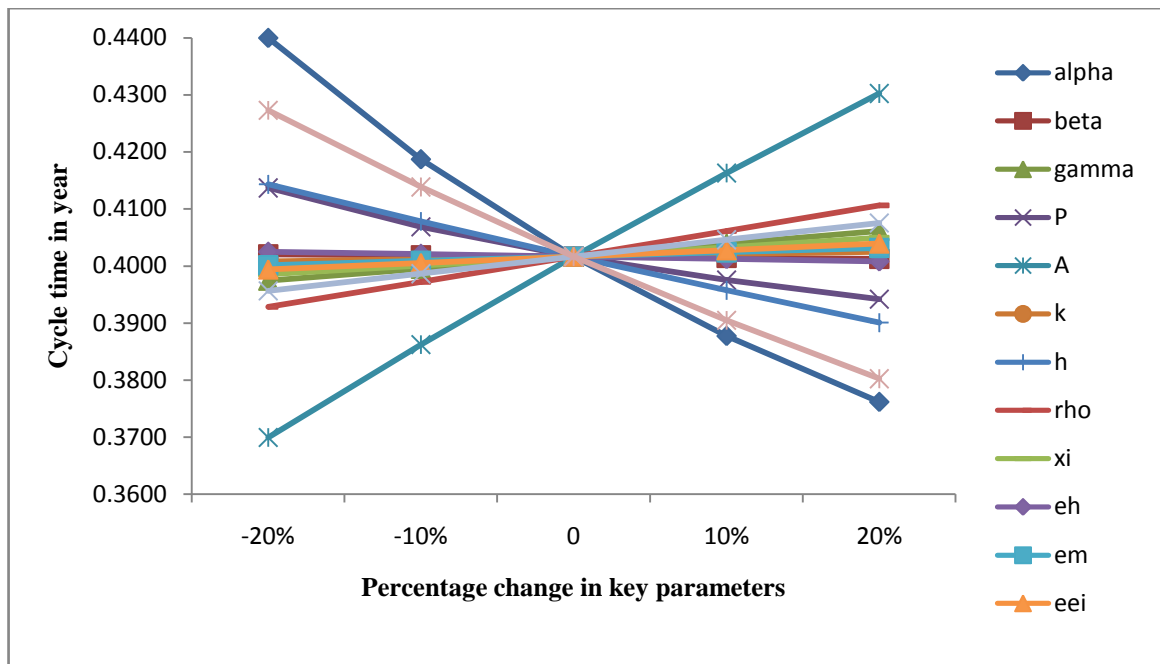


Figure 6.7 Effect of inventory parameters on cycle time

- The analysis shows that how the changes in parameters can affect on the cycle time. Cycle time increase significantly when γ , A , ρ and ξ are increases but the parameters α , P , h and e_{dp} increases the cycle time decreases. Remaining parameters changes effect is minor in cycle time.

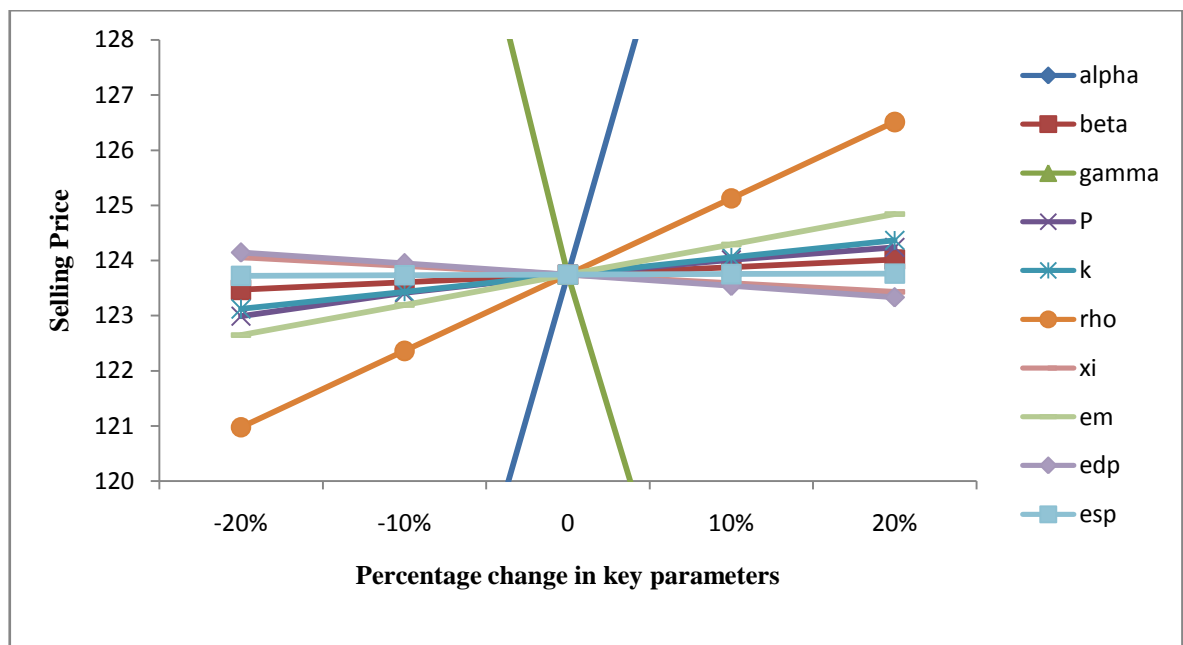


Figure 6.8 Effect of inventory parameters on selling price

- We noticed the impact of the changes in key parameters on the selling price. It clearly shows that the selling price is very sensitive to the parameters α , γ and ρ . With increase in α , ρ , A , k and e_{ei} , then the selling price increases significantly. Further, the increases in parameters γ , ξ and e_{dp} also result into the decrease in selling price. Selling price has negligible sensitive to the parameters β , h , c_{cap} , η and e_{sp} .

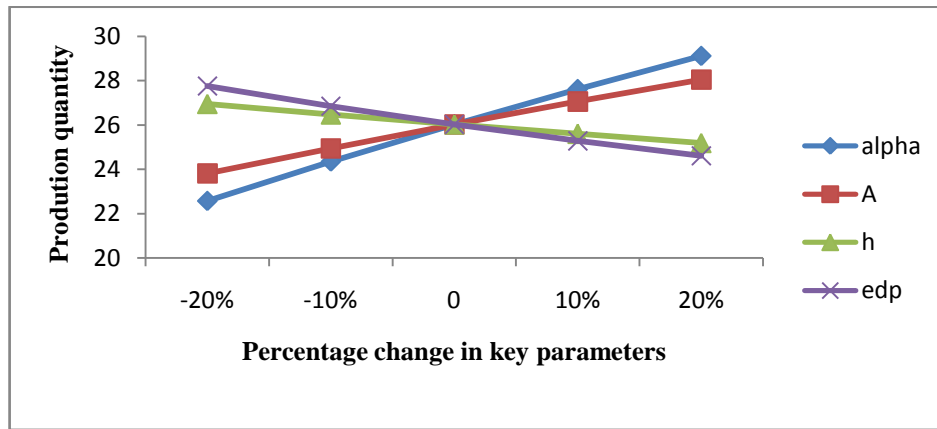


Figure 6.9 Effect of inventory parameters on production quantity

It is observed from Figure 6.9, production quantities are highly influence with changing the parameters α , A , h and e_{dp} . If the increases the value of α , A then production quantity also increase but h and e_{dp} increases the production quantity also decrease, other parameters have a no major effect on production quantity.

6.6 Discussion about managerial insights

The following managerial insights were deduced from the behavioural changes as shown by the sensitivity analysis and mathematical modelling:

- The higher value of carbon emission parameters will result in a decrease in the manufacturer's total profit, which indicates that increased carbon emission will be harmful to the environment and have a negative impact on profit earnings. A decision-maker should attempt to decrease carbon emissions while increasing profit.
- The scale demand and constant coefficient of reduction function indicated that a higher scale demand and a higher value of carbon emission reduction increase the profit with

an increase in the selling price and green investment. It is shown that more investment in green technology results in increased profit.

- The higher value of the selling price may decrease demand. It gives the idea that the manufacturers should try to maintain the selling price so as to increase demand or remain constant and increase profit.
- A higher production rate may decrease profit and increase carbon emissions; on the other hand, a lower production rate may create shortages. Hence, manufacturers should maintain the production rate as per demand.
- As per a government resolution, manufacturers adopted a carbon tax and cap mechanism for sustainability. In our study, it is suggested that the higher value of the carbon cap, the more likely it is to be sold and earn more profit, but the manufacturer pays a higher carbon tax to the government, which may decrease his or her profit. It is recommended that the manufacturer maintain the proper carbon cap and minimize the carbon tax.
- In the study, it is shown that the parameters ξ and η increases then the manufacture's total profit increased and total carbon emission is decreases. This suggests that investing in green technology boosts overall profit and lowers emissions; manufacturers should raise their investments in green technology to increase profit. A carbon cap and tax system increases the company's drive to decrease emissions, such as through investing in environmentally friendly innovations, using alternative sources of energy, or revamping the supply chain network. Manufacturer should be reducing the inventory cost for gain more profit. From the above analysis, it is clear that the higher value of ordering cost, purchase cost, holding cost, production cost are negatively proportional to total profit. Hence, a manufacturer must keep the lower rate of inventory cost parameter.
- By utilizing the proposed model, a decision maker can undoubtedly decide optimum selling price to accomplish a margin in profit. An optimization in the selling price gives an escalation in the demand of the customers and, thusly, to the total profit.
- In terms of business, the perishable product's duration is more significant. A product with an extended lifespan may be beneficial for making greater profits since the seller will have more time to sells the product. Therefore, the management should select the product with the longest lifespan.

6.7 Conclusion

In this chapter, we extend the study of Mishra et al. [24] by considering demand as a carbon reduction function and selling price dependent; and a sustainable inventory model for managing perishable products, in which the product's deterioration fluctuates with time and is determined by its expiration date. Higher investment in green technology and an increase in the carbon reduction rate cause increased market demand and products depreciate throughout the period, consumers are highly conscious about the product's expiration date. A sustainable carbon tax and cap-based production model were developed for a controllable carbon emission rate by investing in green technology initiatives, and the role of green technology investment was identified. The manufacturer's optimal replenishment cycle time, optimal green investment cost, and optimal selling price have been determined. The objective of the chapter is to maximize the manufacturer's total profit at the optimal value of decision variables. Classical optimization method is used to find global maximum solutions. Sensitivity analysis is done for certain important factors that would be useful in developing company strategies, and it shows that the inventory system benefits more from lower manufacturing costs. Some of the key outcomes derived as a investigations, like a green investment that helps to reduce emissions, product longevity that helps to increase profits, maintaining the carbon cap and possibly lowering the carbon tax, maintaining the rate of production, optimized the manufacturers profit; etc. and other key finding mentioned as a significance of this chapter in managerial insights Section 6.6. This model can be useful for the pharmaceutical, chemical and/or pesticides manufacturing industries. This model was developed for perishable products but product obsolescent cost is not considered, rate of production of products may not be constant which depends upon product characteristics, which are the limitations of this chapter.

Additionally, this chapter has several potential extensions. For the future study, the model will be extended for the different payment schemes, and another extension will use preservation technology to reduce the effect of deterioration. Two echelon supply chain models will be developed as a further study by considering green investments and stochastic or product quality-based demand.

CHAPTER-7

Optimal Green Investments and Replenishment Decisions in Vendor Managed Inventory System for Non Instantaneous Deteriorating Products with Partially Backordering

7.1 Introduction

In the actual economy of enterprise, there are times when a producer, a vendor, and a buyer or merchant would like to establish a permanent collaborative partnership as an integrated system to obtain a comfortable, stable way to generate supply and demand for products along with dependability to acquire the most possible profit from one another. It has been effective to put a number of supply chain partner cooperation and coordination initiatives into action. Vendor-managed inventory (VMI) is an ensemble effort that has been proven to increase both the efficiency and adaptability of the supply chain, both conceptually and operationally[246],[247]. All inventory decisions are made by the supplier as a vendor rather than the buyer in a vendor-managed inventory (VMI) system. The effectiveness of an inventory policy becomes sensitive when carbon emissions are taken into consideration. The shipment of products, storage processes, and deterioration of products are to be considered basic contributors to an increase in atmospheric carbon emissions. The importance of integrating sustainable development into inventory and transportation processes has been highlighted because, during the delivery of inventory, the unitization of fuel in vehicles plays a vital role in producing carbon emissions and has

gained prominence in recent times. Green initiatives and their promotion have grown into an important challenge for businesses to improve their environmental consciousness as a result of the growing global awareness of conservation concerns and the associated developments. Investing in green technologies could help lessen the effects of carbon emissions. In past literature, the research work regarding inventory models of VMI systems and traditional inventory systems is scarcely taken together for various factors like 'non-instantaneous deteriorating products, carbon emission-green investment policies, and shortages permitted at the buyer's side with partial backlog' . Keeping all these in mind, this chapter develops a two-level supply chain with a single vendor and buyer, a single product that non-instantaneously deteriorates, and demand for products that are a green investment and their promotion dependent. Therefore, this chapter analyzes the entire cost of the VMI scheme with the total cost of each individual management system in order to investigate inventory modelling of non-instantaneous deteriorating products with a green investment policy and partial backlog shortages from the buyer side, in an effort to close this research gap. The result shows that carbon emission reduction is directly proportional to green technology investment, while green technology promotion increases demand. The objective of this study is to minimize the total cost per time unit of the supply chain and carbon emission cost with respect to optimal green investment cost, replenishment cycle time, and order quantity when a decision-maker takes benefit of the opportunity to promote their sustainable performance. For the proposed model's validation, numerical examples and sensitivity analyses are given. The results show that the VMI model is superior to the traditional supply chain model and that by utilizing green technology, the carbon emission level in the VMI system is lower than it is in the traditional supply chain model.

7.2 Notations and Assumptions

The design of the proposed chapter contains the following notations and assumptions.

7.2.1 Notations

Parameters

A_b	Buyer's ordering cost (in ₹/order)
A_s	Supplier's ordering cost (in ₹/order)

C_h	Product holding cost of buyer (₹/unit)
C_d	Product deterioration cost of buyer (₹/unit)
v	Promotional level
e_h	Carbon emission cost during inventory storage (₹/kilogram)
e_d	Carbon emission cost due to deterioration nature of product (₹/kilogram)
e_{T1}	Carbon emission cost produced by the vehicle (₹/kilogram)
e_{T2}	Extra carbon emission cost for transport per unit item (₹/unit/ kilogram)
t_r	Number of trips of vehicle
C_{fT}	Fix transportation cost (₹/shipment)
C_{VT}	Variable transportation cost, same as fuel price
d	Distance travelled by vehicle from supplier to retailer and retailer to customer(kilometre)
F_{ue}	Fuel utilized when vehicle is empty (litre/distance kilometre)
F_{up}	Extra fuel utilized of the vehicle (litre/distance kilometre/ton payload)
W	Product weight (kilogram)
b_1	Backorder cost (₹/unit/unit time)
l_1	cost of lost sales (₹/unit)
t_d	Non deterioration period (year)
θ	Constant rate of deterioration
Q	Ordering quantity per cycle time, where $Q = IM + IB$
δ	Backlogging parameter, $0 < \delta < 1$

Decision variables

g	Green technology investment cost (₹/unit/cycle time) for traditional model
t_1	Positive cycle time (in year) for traditional model
t_2	Shortage period (in year) , $0 \leq t \leq t_2$, $t_1 + t_2 = T$, for traditional model
g_V	Green technology investment cost (₹/unit/cycle time) for VMI model
t_{IV}	Positive cycle time (in year) for VMI model

t_{2V}	Shortage period (in year) , $t_{1V} + t_{2V} = T_V$ for VMI system
Q^*	Ordering quantity per cycle time, for traditional model
Q_V^*	Ordering quantity per cycle time,in VMI system

Objective functions

$TC(t_1, t_2, g)$	Total inventory cost of individual supply chain system
$TC_V(t_{1V}, t_{2V}, g_V)$	Total inventory cost of VMI system

Other expressions and functions

$I_1(t)$	Inventory level during $0 \leq t \leq t_d$
$I_2(t)$	Inventory level during $t_d \leq t \leq t_1$
$I_3(t)$	Inventory level during shortages, $0 \leq t \leq t_2$, $t_1 + t_2 = T$
$R(g, v)$	Demand function of green investment and it's promotional level
TCB	Buyer's total cost for traditional model
TCB_V	Buyer's total cost for VMI model
TCS	Supplier's total cost for traditional model
TCS_V	Supplier's total cost for VMI model
IM	Maximum positive inventory level per cycle time
IB	Maximum Shortage level per cycle time
$f(g)$	The fraction of reduction of average emission, which is a function of green investments
\hat{C}	Total carbon emission cost before investing in green technology (₹/cycle)
\hat{C}_g	Total carbon emission cost after investing in green technology (₹/cycle)

7.2.2 Assumptions

1. Single vendor and single buyer is considered for modelling of inventory system.
2. Inventory modelling developed for a single non-instantaneous deteriorating product.
3. The deterioration rate of product is non instantaneous. i.e. inventory time $t = 0$ to $t = t_d$ there is no effect of deterioration on products but after time $t = t_d$ to $t = t_1$ product affect by constant rate of deterioration.

4. Replacement or repairing not allowed for deteriorated products.
5. The product's demand is influenced by green investments and its promotional level during the period when inventory is available, and the demand rate is unaffected and remains constant by green investments and promotion levels during the period when there are shortages.
6. Investments in green technology have a positive impact on demand. Consumers become more environmentally conscious as a result of increased green investment promotion.(Zanoni et al. [197], Xia et al.[200], Hasan et al.[202]).
7. The retailer or supplier invests in green technology for sustainability over a certain time frame without raising the unit price of the product.

$$8. \text{ Demand function defined as } R(g, v) = \begin{cases} \alpha + \beta f(g) + \gamma v; & 0 \leq t \leq t_d \\ \alpha + \beta f(g) + \gamma v; & t_d \leq t \leq t_1, \\ \alpha & ; 0 \leq t \leq t_2 \end{cases}$$

Where $\alpha > 0$ is constant market demand, $0 < \beta < 1$ is coefficient of $f(g)$ and $0 < \gamma < 1$ constant coefficient of promotion level v respectively.

9. Rate of replenishment is to be infinite. The lead time is zero or negligible.
10. There is an infinite planning horizon for whole system.
11. In the traditional inventory model, the promotion cost is included with the buyer's ordering costs. In the vendor-managed inventory model, the promotion cost is included in the supplier's ordering costs.
12. Carbon is generated as a result of a number of activities (Hovelaque and Bironneau[168]). We considered some of the factors, such as carbon emissions from the storage facility, the deterioration effects, and the transportation used to move inventory from the supplier to the retailer's warehouse. The delivery distance and emission rate per unit distance are the sole factors affecting emissions, as the vehicle can carry the entire order quantity .(Bonney and Jaber[170],Daryanto et al.[188]).
13. The fuel consumption rate depends upon truckloads.(Hua et al.[166],Mashud[189]).
14. The green technology is used to reduce the carbon emission from the transportation and other inventory operations like storage of inventories, deterioration of inventories etc. To reduce the effect of carbon emission, investment in green technology to be considered. The fraction of reduction of average emission is $f(g) = \xi \left(\frac{\eta g}{1 + \eta g} \right)$; where, $0 < \xi < 1$ is the fraction of carbon emission after investing in green technology,

$\eta > 0$ efficiency of greener technology in reducing emission and $g > 0$ is the green investment cost. Notice that $g \rightarrow 0 \Rightarrow f(g) \rightarrow 0$, $g \rightarrow \infty \Rightarrow f(g) \rightarrow \xi$ and $f(g)$ is continuously differentiable with $f'(g) > 0, f''(g) < 0$. (Bhavani et al.[187])

15. In the VMI system, the vendor is responsible for green technology investment. In a traditional inventory system, buyers need to invest in green technology.
16. Partially backordering shortages are at buyer side with backlogging rate is $S(x) = e^{-\delta x}$ with $0 < \delta < 1$ up to next replenishment, where x denotes the waiting time of the buyer. Shortages are not allowed at vendor side.(Abad[33], [85])

7.3 Mathematical formulation

The current study considers the inventory model that incorporates the consideration of stock available at the beginning of cycle time. This section, outlines the progression of the conventional buyer's model and vendor managed inventory(VMI) model in the context of non-instantaneous deteriorating products. The construction of the models is based on notations and assumptions outlined in the section 7.2. Within the designated interval $[0, t_d]$ inventory level experiences a reduction due to demand and during interval $[t_d, t_1]$ inventory level depletes due to the combined effects of demand and deterioration and inventory level goes to zero. Following this, once the inventory level reaches zero, shortages begin to occur and be gradually accumulate in $[0, t_2]$. The aforementioned inventory systems undergo a repetitive process. The inventory level pattern is depicted in Figure 7.1.

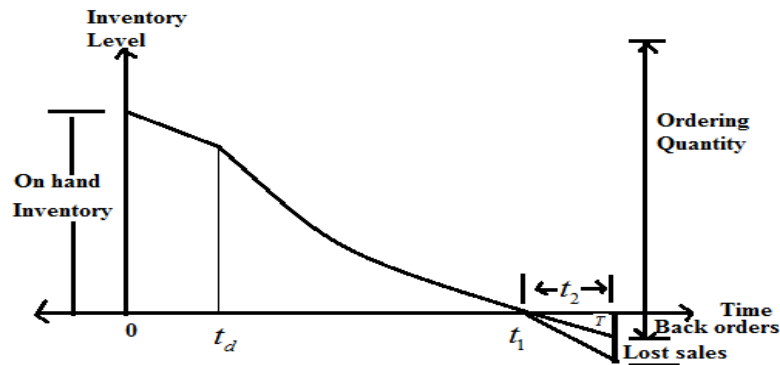


Figure 7.1 The graphical representation for the inventory system

The differential equations (7.1) and (7.2) governing the status of inventory level of the product at time t over the period $[0, t_d]$ and $[t_d, t_1]$ respectively, are as follows:

$$\frac{dI_1(t)}{dt} = -(\alpha + \beta f(g) + \gamma v); 0 \leq t \leq t_d \quad (7.1)$$

$$\frac{dI_2(t)}{dt} + \theta I_2(t) = -(\alpha + \beta f(g) + \gamma v); t_d \leq t \leq t_2 \quad (7.2)$$

The solution of (7.1) and (7.2) is respectively for the conditions $I_1(0) = IM$ and $I_2(t_1) = 0$, given by,

$$I_1(t) = IM - \left(\alpha + \beta \xi \left(\frac{\eta g}{1 + \eta g} \right) + \gamma v \right) t \quad (7.3)$$

$$I_2(t) = \frac{(e^{-(t-t_1)\theta} - 1)}{\theta} \left(\alpha + \beta \xi \left(\frac{\eta g}{1 + \eta g} \right) + \gamma v \right) \quad (7.4)$$

Since $I_1(t) = I_2(t)$ at $t = t_d$, we have maximum positive inventory level is,

$$IM = \left(\frac{(e^{-(t_d-t_1)\theta} - 1)}{\theta} + t_d \right) \left(\alpha + \beta \xi \left(\frac{\eta g}{1 + \eta g} \right) + \gamma v \right) \quad (7.5)$$

From the (7.3),

$$I_1(t) = \left(\frac{(e^{-(t_d-t_1)\theta} - 1)}{\theta} + t_d - t \right) \left(\alpha + \beta \xi \left(\frac{\eta g}{1 + \eta g} \right) + \gamma v \right) \quad (7.6)$$

Shortages occur when the inventory level is approaches to zero at time t_1 . During the stock out phase, some buyers may be ready to wait for a delivery delay, whilst others may depart for another vendor due to an urgent requirement. The inventory level for a customer who wishes to make a purchase during $[0, t_2]$, is determined by the following differential equation,

$$\frac{dI_3(t)}{dt} = -\alpha e^{-\delta(t_2-t)}, 0 \leq t \leq t_2 \text{ with } I_2(0) = 0 \quad (7.7)$$

The solution of the differential equation (7.7) is given by,

$$I_3(t) = \frac{\alpha e^{-\delta t_2} (1 - e^{\delta t})}{\delta}, 0 \leq t \leq t_2 \quad (7.8)$$

The maximum back order units are,

$$IB = -I_3(t_2) = \frac{\alpha(1 - e^{-\delta t_2})}{\delta} \quad (7.9)$$

Thus, the ordering quantity per order over the replenishment cycle time from (7.5) and (7.9) can be derived as,

$$Q = IM + IB = \left(\frac{(e^{-(t_d - t_1)\theta} - 1)}{\theta} + t_d \right) \left(\alpha + \beta \xi \left(\frac{\eta g}{1 + \eta g} \right) + \gamma v \right) + \frac{\alpha(1 - e^{-\delta t_2})}{\delta} \quad (7.10)$$

Our objective is to minimize the total cost of the supply chain, the following cost elements to be calculated:

$$\text{Buyer's ordering cost (OB): } \frac{A_b}{t_1 + t_2} \quad (7.11)$$

$$\text{Supplier's ordering cost (OS): } \frac{A_s}{t_1 + t_2} \quad (7.12)$$

The inventory holding cost during the positive cycle time:.,

$$HC = \frac{C_h}{t_1 + t_2} \left[\int_0^{t_d} I_1(t) dt + \int_{t_d}^{t_1} I_2(t) dt \right] \quad (7.13)$$

Cost due to deterioration of product:

$$DC = \frac{C_d}{t_1 + t_2} \left[I_2(t_d) - \int_{t_d}^{t_1} (\alpha + \beta f(g) + \gamma v) dt \right] \quad (7.14)$$

$$\text{Green technology investment cost: } GTC = \frac{g(t_1 + t_2)}{(t_1 + t_2)} \quad (7.15)$$

Shortage cost due to back order:

$$BC = \frac{b_1}{t_1 + t_2} \int_0^{t_2} (-I_3(t)) dt = \frac{b_1}{t_1 + t_2} \frac{\alpha(1 - \delta t_2 e^{-\delta t_2} - e^{-\delta t_2})}{\delta^2} \quad (7.16)$$

$$\text{Cost due to lost sale: } LS = \frac{l_1}{t_1 + t_2} \int_0^{t_2} \alpha(1 - e^{-\delta(t_2 - t)}) dt = \frac{l_1}{t_1 + t_2} \frac{\alpha(\delta t_2 + e^{-\delta t_2} - 1)}{\delta} \quad (7.17)$$

Here variable transportation cost C_{VT} is multiply by vehicle fuel consumption F_{ue} . $2d$ mentioned two sided vehicle trips. The extra vehicle fuel utilization F_{up} is needed for one trip when the truck is loaded with the products Q units with weight per unit is W kilogram, so the distance d is multiply with C_{VT} . Total transportation cost depends on number of trips for deliver the product from supplier to retailer is,

$$TNC = \frac{t_r}{t_1 + t_2} [C_{fT} + (2dC_{VT}F_{ue} + dC_{VT}F_{up}WQ)] \quad (7.18)$$

For the calculate the carbon emission cost of the system following components to be consider,

The emission cost from holding operations:

$$HC_e = \frac{e_h}{t_1 + t_2} \left[\int_0^{t_d} I_1(t) dt + \int_{t_d}^{t_1} I_d(t) dt \right] \quad (7.19)$$

The emission cost depends on the delivery quantity and distance travelled by vehicle. The emission cost during delivered the product from supplier to retailer's warehouse:

$$TNC_e = \frac{t_r}{t_1 + t_2} [2de_{T1} + de_{T2}Q] \quad (7.20)$$

The emission cost due to deterioration of product:,

$$DC_e = \frac{e_d}{t_1 + t_2} \left[I_2(t_d) - \int_{t_d}^{t_1} (\alpha + \beta f(g) + \gamma t) dt \right] \quad (7.21)$$

Total carbon emission from (7.19) to (7.21) before investing in green technology is,

$$\hat{C} = HC_e + DC_e + TNC_e \quad (7.22)$$

By investing in green technology, the carbon emission cost is,

$$\hat{C}_g = \hat{C}(1 - f(g)) \quad (7.23)$$

Now, next to we developed traditional inventory model and vendor managed inventory model as per the notations and assumptions, and derive the results for both systems.

7.3.1 Green traditional inventory model

This section covered the integrated economic and environmental ideas in developing the inventory model. So the traditional inventory model is named a green traditional inventory model.

In the green tradition inventory system, total cost of the supplier and buyer before applying vendor inventory system are respectively given by,

$$TCB = OB + HC + DC + GTC + BC + LS + TNC + \hat{C}_g \quad (7.24)$$

Note that (7.24) can be re written as ,

$$TCB = \frac{\mu(t_1, t_2, g)}{t_1 + t_2} \quad (7.25)$$

Where,

$$\mu(t_1, t_2, g) = \left\{ \begin{aligned} & A_b + C_h \left[\left(\frac{(e^{-(t_d-t_1)\theta} - 1)}{\theta} (t_d) + \frac{t_d^2}{2} + \frac{e^{-(t_d-t_1)\theta} - 1 + \theta(t_d-t_1)}{\theta^2} \right) R(g, v) \right] \\ & + C_d \left[\left(\frac{(e^{-(t_d-t_1)\theta} - 1)}{\theta} - t_1 + t_d \right) D(g, v) \right] + g(t_1 + t_2) + \frac{b_1 \alpha (1 - \delta t_2 e^{-\delta t_2} - e^{-\delta t_2})}{\delta^2} \\ & + \frac{l_1 \alpha (\delta t_2 + e^{-\delta t_2} - 1)}{\delta} \\ & + t_r \left[C_{fT} + \left(2dC_{VT} F_{ue} + dC_{VT} F_{up} W \left\{ \left(\frac{(e^{-(t_d-t_1)\theta} - 1)}{\theta} + t_d \right) R(g, v) + \frac{\alpha(1 - e^{-\delta t_2})}{\delta} \right\} \right) \right] \\ & \left(e_h \left[\left(\frac{(e^{-(t_d-t_1)\theta} - 1)}{\theta} (t_d) + \frac{t_d^2}{2} + \frac{e^{-(t_d-t_1)\theta} - 1 + \theta(t_d-t_1)}{\theta^2} \right) R(g, v) \right] \right. \\ & + e_d \left[\left(\frac{(e^{-(t_d-t_1)\theta} - 1)}{\theta} - t_1 + t_d \right) D(g, v) \right] \left. \left(1 - \xi \left(\frac{\eta g}{1 + \eta g} \right) \right) \right. \\ & \left. + t_r \left[2de_{T1} + de_{T2} \left\{ \left(\frac{(e^{-(t_d-t_1)\theta} - 1)}{\theta} + t_d \right) R(g, v) + \frac{\alpha(1 - e^{-\delta t_2})}{\delta} \right\} \right] \right) \end{aligned} \right\}$$

And $TCS = \frac{A_s}{t_1 + t_2}$

For the buyer, the optimization problem in tradition supply chain system is,

$$\underset{t_1, t_2, g}{\text{Minimize}} TCB(t_1, t_2, g),$$

subject to $0 < t < t_1 + t_2 = T$

The methodology for prove the convexity of cost function adopted as per Sana et al.[248] and Soni[134], The necessary conditions for the total average cost of the buyer $TCB(t_1, t_2, g)$ to be the minimum are,

$$\frac{\partial TCB(t_1, t_2, g)}{\partial t_1} = 0, \frac{\partial TCB(t_1, t_2, g)}{\partial t_2} = 0 \& \frac{\partial TCB(t_1, t_2, g)}{\partial g} = 0 \quad (7.26)$$

From (7.25), we have

$$\frac{\partial TCB(t_1, t_2, g)}{\partial t_1} = -\frac{\mu(t_1, t_2, g)}{(t_1 + t_2)^2} + \frac{1}{t_1 + t_2} \frac{\partial \mu(t_1, t_2, g)}{\partial t_1} \quad (7.27)$$

$$\frac{\partial^2 TCB(t_1, t_2, g)}{\partial t_1^2} = \frac{2\mu(t_1, t_2, g)}{(t_1 + t_2)^3} - \frac{1}{(t_1 + t_2)^2} \frac{\partial \mu(t_1, t_2, g)}{\partial t_1} + \frac{1}{t_1 + t_2} \frac{\partial^2 \mu(t_1, t_2, g)}{\partial t_1^2} \quad (7.28)$$

$$\frac{\partial TCB(t_1, t_2, g)}{\partial t_2} = -\frac{\mu(t_1, t_2, g)}{(t_1 + t_2)^2} + \frac{1}{t_1 + t_2} \frac{\partial \mu(t_1, t_2, g)}{\partial t_2} \quad (7.29)$$

$$\frac{\partial^2 TCB(t_1, t_2, g)}{\partial t_2^2} = \frac{2\mu(t_1, t_2, g)}{(t_1 + t_2)^3} - \frac{1}{(t_1 + t_2)^2} \frac{\partial \mu(t_1, t_2, g)}{\partial t_2} + \frac{1}{t_1 + t_2} \frac{\partial^2 \mu(t_1, t_2, g)}{\partial t_2^2} \quad (7.30)$$

From the necessary conditions,

$$\frac{\partial TCB(t_1, t_2, g)}{\partial t_1} = 0 = \frac{\partial TCB(t_1, t_2, g)}{\partial t_2} \quad (7.31)$$

$$\mu(t_1, t_2, g) = (t_1 + t_2) \frac{\partial \mu(t_1, t_2, g)}{\partial t_1} \quad (7.32)$$

$$\mu(t_1, t_2, g) = (t_1 + t_2) \frac{\partial \mu(t_1, t_2, g)}{\partial t_2} \quad (7.33)$$

$$\frac{\partial^2 TCB(t_1, t_2, g)}{\partial t_1^2} = \frac{1}{t_1 + t_2} \frac{\partial^2 \mu(t_1, t_2, g)}{\partial t_1^2} \quad (7.34)$$

$$\frac{\partial^2 TCB(t_1, t_2, g)}{\partial t_2^2} = \frac{1}{t_1 + t_2} \frac{\partial^2 \mu(t_1, t_2, g)}{\partial t_2^2} \quad (7.35)$$

$$\frac{\partial \mu(t_1, t_2, g)}{\partial t_1} = \frac{\partial \mu(t_1, t_2, g)}{\partial t_2} \quad (7.36)$$

$$\frac{\partial \mu(t_1, t_2, g)}{\partial t_1} = \left\{ \begin{aligned} & C_h \left[\left((e^{-(t_d - t_1)\theta})(t_d) + \frac{e^{-(t_d - t_1)\theta} - 1}{\theta} \right) R(g, v) \right] \\ & + C_d \left[(e^{-(t_d - t_1)\theta} - 1) D(g, v) \right] + g \\ & + t_r \left[\left(dC_{VT} F_{up} W \left\{ (e^{-(t_d - t_1)\theta}) R(g, v) \right\} \right) \right] \\ & \left(e_h \left[\left((e^{-(t_d - t_1)\theta})(t_d) + \frac{e^{-(t_d - t_1)\theta} - 1}{\theta} \right) R(g, v) \right] \right. \\ & \left. + e_d \left[(e^{-(t_d - t_1)\theta} - 1) R(g, v) \right] \right. \\ & \left. + t_r \left[de_{T2} \left\{ (e^{-(t_d - t_1)\theta}) R(g, v) \right\} \right] \right) \left(1 - \xi \left(\frac{\eta g}{1 + \eta g} \right) \right) \end{aligned} \right\} \quad (7.37)$$

$$\frac{\partial \mu(t_1, t_2, g)}{\partial t_2} = \left\{ \begin{aligned} & g + \frac{b_1 \alpha (\delta^2 t_2 e^{-\delta t_2} - 2 \delta e^{-\delta t_2})}{\delta^2} + l_1 \alpha (1 - e^{-\delta t_2}) \\ & + t_r \left[\alpha e^{-\delta t_2} \right] + \left(t_r \left[\alpha e^{-\delta t_2} \right] \right) \left(1 - \xi \left(\frac{\eta g}{1 + \eta g} \right) \right) \end{aligned} \right\} \quad (7.38)$$

$$\frac{\partial^2 \mu(t_1, t_2, g)}{\partial t_1^2} = \left\{ \begin{aligned} & C_h \left[\left((\theta e^{-(t_d - t_1)\theta})(t_d) + e^{-(t_d - t_1)\theta} \right) R(g, v) \right] \\ & + C_d \left[\left(\theta e^{-(t_d - t_1)\theta} \right) R(g, v) \right] + \\ & + t_r \left[\left(dC_{VT} F_{up} W \left\{ (\theta e^{-(t_d - t_1)\theta}) R(g, v) \right\} \right) \right] \\ & \left(e_h \left[\left((\theta e^{-(t_d - t_1)\theta})(t_d) + e^{-(t_d - t_1)\theta} \right) R(g, v) \right] \right. \\ & \left. + e_d \left[\left(\theta e^{-(t_d - t_1)\theta} \right) R(g, v) \right] \right. \\ & \left. + t_r \left[de_{T2} \left\{ (\theta e^{-(t_d - t_1)\theta}) R(g, v) \right\} \right] \right) \left(1 - \xi \left(\frac{\eta g}{1 + \eta g} \right) \right) \end{aligned} \right\} \quad (7.39)$$

$$\frac{\partial^2 \mu(t_1, t_2, g)}{\partial t_2^2} = \left\{ \begin{aligned} & b_1 \alpha (-\delta t_2 e^{-\delta t_2} - e^{-\delta t_2}) + l_1 \alpha (\delta e^{-\delta t_2}) + t_r [-\delta \alpha e^{-\delta t_2}] \\ & + \left(t_r [-\delta \alpha e^{-\delta t_2}] \right) \left(1 - \xi \left(\frac{\eta g}{1 + \eta g} \right) \right) \end{aligned} \right\} \quad (7.40)$$

$$\frac{\partial^2 \mu(t_1, t_2, g)}{\partial t_1 \partial t_2} = 0 = \frac{\partial^2 \mu(t_1, t_2, g)}{\partial t_2 \partial t_1} \quad (7.41)$$

From (7.37), $M(t_1) = N(t_2)$ where,

$$M(t_1) = \left\{ \begin{aligned} & \left(\begin{aligned} & C_h \left[\left((e^{-(t_d - t_1)\theta})(t_d) + \frac{e^{-(t_d - t_1)\theta} - 1}{\theta} \right) \right] \\ & + C_d \left[(e^{-(t_d - t_1)\theta} - 1) \right] \\ & + t_r \left[(dC_{VT} F_{up} W \{ (e^{-(t_d - t_1)\theta}) \}) \right] \end{aligned} \right) R(g, v) \\ & + \left(\begin{aligned} & e_h \left[\left((e^{-(t_d - t_1)\theta})(t_d) + \frac{e^{-(t_d - t_1)\theta} - 1}{\theta} \right) \right] \\ & + e_d \left[(e^{-(t_d - t_1)\theta} - 1) \right] \\ & + t_r \left[de_{T2} \{ (e^{-(t_d - t_1)\theta}) \} \right] \end{aligned} \right) (1 - f(g)) R(g, v) \end{aligned} \right\} \text{ and} \quad (7.42)$$

$$N(t_2) = \left\{ \begin{aligned} & \frac{b_1 \alpha (\delta^2 t_2 e^{-\delta t_2} - 2\delta e^{-\delta t_2})}{\delta^2} + l_1 \alpha (1 - e^{-\delta t_2}) \\ & + t_r [\alpha e^{-\delta t_2}] \\ & + \left(t_r [\alpha e^{-\delta t_2}] \right) \left(1 - \xi \left(\frac{\eta g}{1 + \eta g} \right) \right) \end{aligned} \right\} \quad (7.43)$$

$$\frac{dN(t_2)}{dt_2} = (b_1(3 - t_2\delta) + \delta l_1 - \delta t_r - (\delta t_r)(1 - f(g))) \alpha e^{-\delta t_2}$$

$$\frac{dN(t_2)}{dt_2} \geq 0, \text{ if } t_2 \leq \frac{3b_1 + \delta l_1 - \delta t_r - \delta t_r(1 - f(g))}{\delta b_1} = \tilde{t}_2$$

$$\text{and } \frac{dN(t_2)}{dt_2} \leq 0, \text{ if } t_2 \geq \frac{3b_1 + \delta l_1 - \delta t_r - \delta t_r(1 - f(g))}{\delta b_1} = \tilde{t}_2$$

Hence $N(t_2)$ is monotonic increasing function of $t_2 \in (0, \tilde{t}_2)$ and is monotonic decreasing function of $t_2 \in (\tilde{t}_2, \infty)$. Maximum value of $N_{max} = N(\tilde{t}_2)$ and as $t_1 \rightarrow \infty$, $M(t_1)$ is increasing function of t_1 then $\exists \tilde{t}_1$ unique such that $M(\tilde{t}_1) = N_{max}$. It is conclude that $t_2^* \in (0, \tilde{t}_2)$ there exist unique $t_1^* \in (0, \tilde{t}_1)$ such that $M(t_1^*) = N(t_2^*)$. Therefore t_1^* uniquely determined as a function of t_2^* .

Theorem: 7.1 *For the any positive fix value of g , buyer's total average cost function $TCB(t_1, t_2, g)$ is convex and reaches its global minimum at (t_1^*, t_2^*) .*

Proof: From (7.39) and (7.40), it shown that $\frac{\partial^2 \mu(t_1, t_2, g)}{\partial t_1^2} > 0$ and $\frac{\partial^2 \mu(t_1, t_2, g)}{\partial t_2^2} > 0$ and from the (7.34), (7.35) and (7.41), notice that

$$\left[\frac{\partial^2 TCB(t_1, t_2, g)}{\partial t_1^2} \frac{\partial^2 TCB(t_1, t_2, g)}{\partial t_2^2} - \left(\frac{\partial^2 TCB(t_1, t_2, g)}{\partial t_1 \partial t_2} \right)^2 \right]_{(t_1, t_2) = (t_1^*, t_2^*)} > 0$$

Hence, hessian matrix became negative definite at point $(t_1, t_2) = (t_1^*, t_2^*)$ for the positive fix value of g . So, $TCB(t_1, t_2, g)$ is convex and reaches minimize at (t_1^*, t_2^*) .

Theorem: 7.2 *For the any given (t_1^*, t_2^*) , buyer's total cost function $TCB(t_1, t_2, g)$ is convex and approaches its global minimum at $g = g^*$.*

Proof: For the any given (t_1^*, t_2^*) , the necessary condition to obtained $g = g^*$ is $\frac{\partial TCB(t_1, t_2, g)}{\partial g} = 0$ and it is easily verified the sufficient condition, the second order partial

derivative is $\frac{\partial^2 TCB(t_1, t_2, g)}{\partial g^2} > 0$ at $g = g^*$.

Next, we discuss about the green vendor managed inventory system.

7.3.2 Green Vendor Managed Inventory supply chain system

VMI system referred to as the "green VMI system" in this section integrated economic and environmental goals are taken into consideration. In the VMI relationship, the vendor (the supplier) has the accountability of controlling inventory levels at the point of sale by establishing when to replenish and the number of orders by obtaining information about

the whole supply chain. In a vendor-managed inventory system, the supplier bears the costs of the buyer. So the buyer's cost and supplier's cost in vendor managed inventory model is respectively as follows,

$$TCB_V = 0 \text{ and } TCS_V = (OS + OB + HC + DC + GTC + BC + LS + TNC + \hat{C}_g)$$

Total average cost of supply chain in VMI model is

$$TC_V(t_{1V}, t_{2V}, g_V) = TCB_V(t_{1V}, t_{2V}, g_V) + TCS_V(t_{1V}, t_{2V}, g_V)$$

$$= \frac{1}{t_{1V} + t_{2V}} \left(\begin{aligned} & A_s + A_b + C_h \left[\int_0^{t_d} I_1(t) dt + \int_{t_d}^{t_{1V}} I_2(t) dt \right] + C_d \left[I_2(t_d) - \int_{t_d}^{t_{1V}} (\alpha + \beta f(g) + \gamma v) dt \right] + g(t_{1V} + t_{2V}) \\ & + b_1 \int_0^{t_{2V}} (-I_3(t)) dt + l_1 \int_0^{t_{2V}} \alpha (1 - e^{-\delta(t_{2V}-t)}) dt + t_r [C_{fT} + (2dC_{VT}F_{ue} + dC_{VT}F_{up}WQ)] \\ & + \left(\begin{aligned} & e_h \left[\int_0^{t_d} I_1(t) dt + \int_{t_d}^{t_{1V}} I_d(t) dt \right] + t_r [2de_{T1} + de_{T2}Q] \\ & + e_d \left[I_2(t_d) - \int_{t_d}^{t_{1V}} (\alpha + \beta f(g) + \gamma t) dt \right] \end{aligned} \right) (1 - f(g)) \end{aligned} \right) \quad (7.44)$$

The optimization problem in green vendor managed inventory supply chain system is,

$$\underset{t_{1V}, t_{2V}, g_V}{\text{Minimize}} TC_V(t_{1V}, t_{2V}, g_V),$$

$$\text{subject to } 0 < t < t_{1V} + t_{2V} = T_V$$

Convexity of total cost function at decision variables and solution procedure followed as per green traditional inventory model discussed in above.

In the next sections we validate the traditional inventory model and VMI model through numerical example and sensitivity analysis.

7.4 Numerical experiments

The following value of parameters in correct units considered as input for the numerical, graphical, and sensitivity analyses of the model.

Example 7.4.1: The values of parameters taken from the different literature and real-world data are suitable for the proposed model.

$A_s = ₹70$ per order, $A_b = ₹30$ per order, $C_h = ₹5$ per unit per year, $C_d = ₹10$ per unit, $v = 15$, $\alpha = 300$, $\beta = 5$, $\gamma = 0.5$, $e_h = ₹5$ per kilogram, $e_d = ₹2$ per kilogram, $e_{T1} = ₹3$ per kilogram, $e_{T2} = ₹1$ per kilogram, $t_r = 2$, $C_{JT} = ₹3$ /kilometer, $C_{VT} = ₹0.1$ /litre, $d = 10$ kilometer, $F_{ue} = 1$ litre/10kilometer, $F_{up} = 2$ litre/10kilometers/ payload, $W = 3$ kilogram/unit, $\eta = 0.8$, $\xi = 0.2$, $t_d = \frac{30}{365}$ year, $\theta = 0.04$, $\delta = 0.05$, $b_1 = ₹80$ /unit, $l_1 = ₹60$ /unit.

Using the above value of parameters and methodology adopted in section 7.4 and solving equations by the mathematical software like maple 18 or matlab or Mathematica, the optimal value of decisions variable mentioned in Table 7.1, Table 7.2 and Table 7.3.

Table 7.1 Optimal result for green VMI model

g_v^* (in ₹)	t_{1v}^* (year)	t_{2v}^* (year)	Q_v^* (units)	TC_v^* (in ₹)	\hat{C}_v^* (in ₹)	\hat{C}_{gv}^* (in ₹)
39.29	0.3258	0.0521	116	9747.76	6701.31	5402.40

Table 7.2 Optimal result for green traditional model

g^* (in ₹)	t_1^* (year)	t_2^* (year)	Q^* (units)	TC^* (in ₹)	\hat{C}^* (in ₹)	\hat{C}_g^* (in ₹)
39.36	0.2642	0.0437	95	9770.96	6724.08	5420.66

Table 7.3 indicated that the increases the value of green investment then total carbon emission cost will decreases and total cost is minimum at the optimum value of green investment cost and cycle time in VMI and traditional model.

Table 7.3 Impact of green investment on total cost and carbon emission

VMI model					Traditional model				
g_v	t_{1v}	t_{2v}	\hat{C}_{gv}	TC_v	g	t_1	t_2	\hat{C}_g	TC
0	0.3253	0.0566	6675.51	10978.51	0	0.2707	0.0483	6694.38	10998.18
20	0.3258	0.0522	5439.28	9765.26	20	0.2644	0.04383	5457.65	9788.32
39.29	0.3258	0.0521	5402.40	9747.76	39.36	0.2642	0.0436	5420.66	9770.96
60	0.3258	0.0520	5388.65	9754.77	60	0.2641	0.0436	5406.98	9778.03
80	0.3258	0.0520	5382.02	9768.16	80	0.2641	0.0436	5400.35	9791.46

← the optimal total cost; bold value indicates the optimal results.

7.4.1 Graphical authentication of objective functions

The convexity performance with respect to decision variables for total cost function is demonstrated as below in Figure 7.2, Figure 7.3, Figure 7.4, Figure 7.5, Figure 7.6 and Figure 7.7.

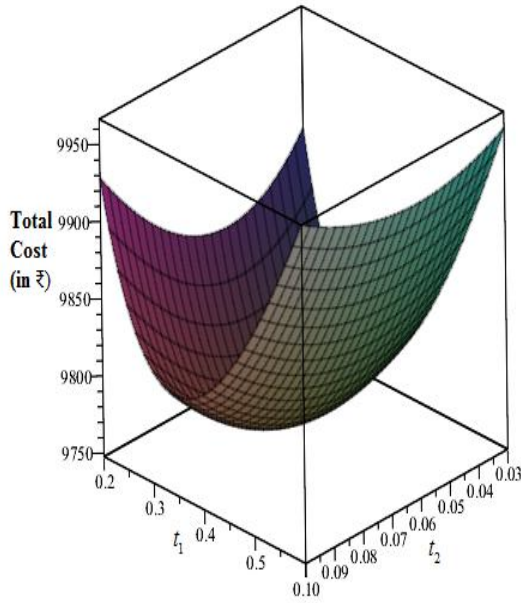


Figure 7.2 Convexity of total cost $TC(g, t_1, t_2)$ with respect to t_1 and t_2

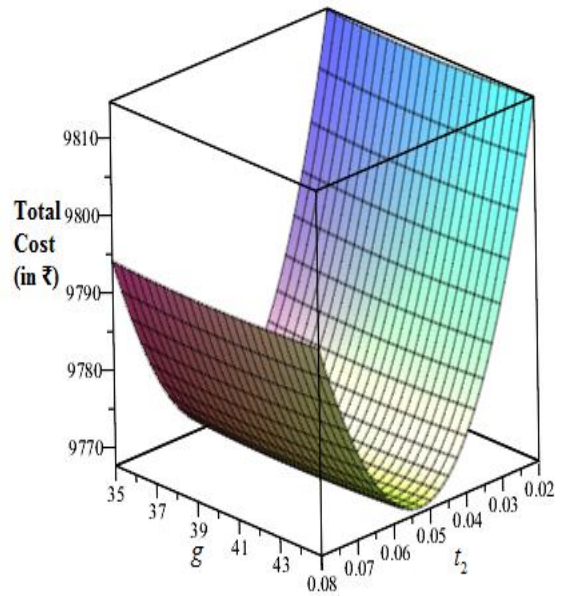


Figure 7.3 Convexity of total cost $TC(g, t_1, t_2)$ with respect to g and t_2

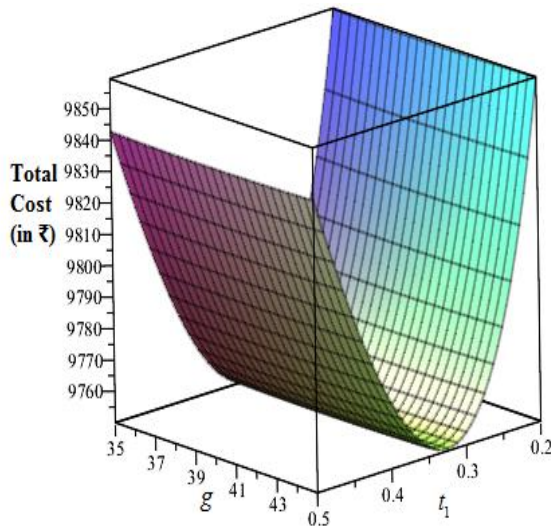


Figure 7.4 Convexity of total cost $TC(g, t_1, t_2)$ with respect to g and t_1

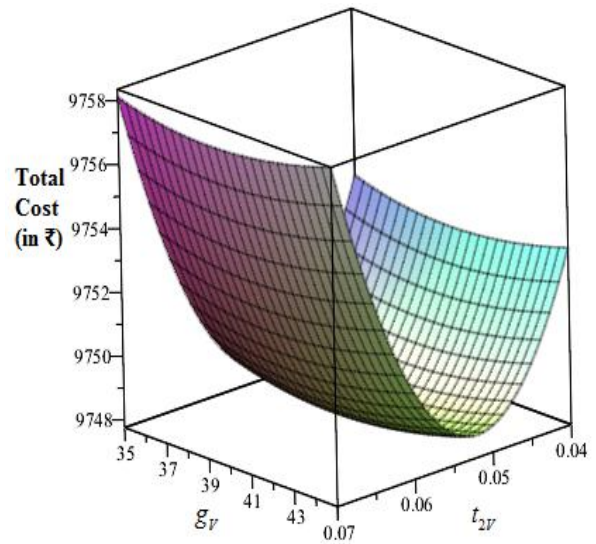


Figure 7.5 Convexity of total cost $TC_v(g, t_1, t_2)$ with respect to g_v and t_{2v}

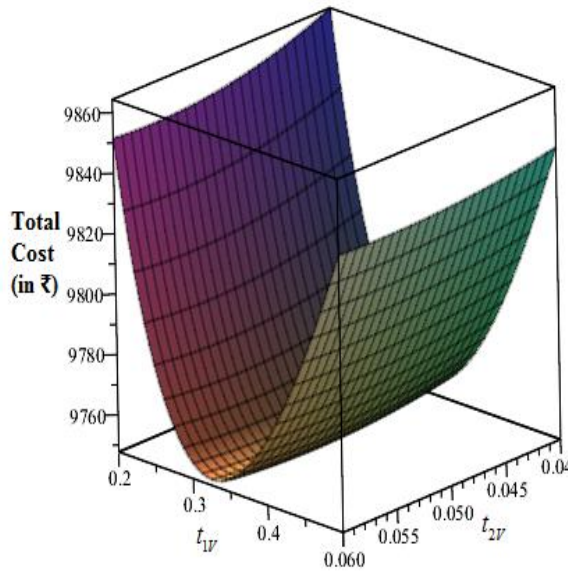


Figure 7.6 Convexity of total cost $TC_V(g, t_1, t_2)$ with respect to t_{1V} and t_{2V}

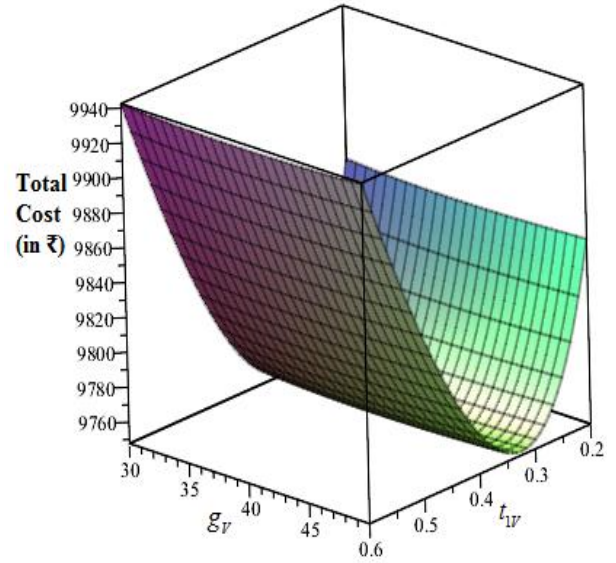


Figure 7.7 Convexity of total cost $TC_V(g, t_1, t_2)$ with respect to g_V and t_{1V}

7.5 Sensitivity analysis and observations

For the proposed model, sensitivity analysis is performed using mathematical software such as Maple 18 or Matlab to determine the feasibility of the model. Parameters by changing -40%, 20%, 20% and 40% in original values as per example taken in section 7.4, we observed the changes in decision variables, and cost function and ordering quantity in vendor managed inventory(VMI) supply chain and traditional supply chain model.

Table 7.4 Sensitivity performance of inventory parameters

Parameters	Value	VMI model					Traditional Model				
		g_V^*	t_{1V}^*	t_{2V}^*	Q_V^*	TC_V^*	g^*	t_1^*	t_2^*	Q^*	TC^*
α	180	30.56	0.41	0.07	91	6141.72	30.64	0.33	0.06	74	6160.03
	240	35.20	0.36	0.06	104	7952.68	35.28	0.29	0.05	85	7973.62
	360	42.98	0.30	0.05	127	11531.51	43.06	0.24	0.04	104	11556.76
	420	46.38	0.28	0.04	138	13306.67	46.45	0.23	0.04	112	13333.83
β	3	39.43	0.33	0.05	116	9737.81	39.50	0.26	0.04	95	9760.97
	4	39.36	0.33	0.05	116	9742.78	39.43	0.26	0.04	95	9765.97
	6	39.22	0.33	0.05	116	9752.71	39.30	0.26	0.04	95	9775.94
	7	39.15	0.33	0.05	117	9757.70	39.23	0.26	0.04	95	9780.93
γ	0.3	39.13	0.33	0.05	116	9670.36	39.21	0.27	0.04	95	9693.38
	0.4	39.21	0.33	0.05	116	9709.16	39.29	0.27	0.04	95	9732.26
	0.6	39.36	0.32	0.05	117	9786.13	39.44	0.26	0.05	95	9809.45

Parameters	Value	VMI model					Traditional Model				
		g_V^*	t_{1V}^*	t_{2V}^*	Q_V^*	TC_V^*	g^*	t_1^*	t_2^*	Q^*	TC^*
	0.7	39.44	0.32	0.06	117	9824.25	39.51	0.26	0.05	95	9847.69
ν	9	39.13	0.33	0.05	116	9670.36	39.21	0.27	0.04	95	9693.38
	12	39.21	0.33	0.05	116	9709.16	39.29	0.27	0.04	95	9732.26
	18	39.36	0.32	0.05	117	9786.13	39.44	0.26	0.05	95	9809.45
	21	39.44	0.32	0.06	117	9824.25	39.51	0.26	0.05	95	9847.69
e_{T1}	0.6	30.91	0.33	0.05	118	7757.84	31.01	0.27	0.04	96	7780.77
	0.8	35.34	0.33	0.05	117	8753.37	35.43	0.27	0.04	95	8776.43
	1.2	42.88	0.32	0.05	116	10741.26	42.95	0.26	0.04	94	10764.59
	1.4	46.20	0.32	0.05	115	11734.03	46.26	0.26	0.05	94	11757.53
e_h	3	39.01	0.36	0.05	125	9673.63	39.11	0.29	0.04	102	9695.2
	4	39.16	0.34	0.05	121	9711.72	39.24	0.28	0.04	98	9734.14
	6	39.41	0.31	0.05	113	9781.93	39.47	0.25	0.04	92	9805.9
	7	39.51	0.30	0.05	109	9814.43	39.57	0.24	0.05	89	9839.11
e_d	1.2	39.29	0.33	0.05	117	9747.12	39.36	0.26	0.04	95	9770.32
	1.6	39.29	0.33	0.05	117	9747.46	39.36	0.26	0.04	95	9770.66
	2.4	39.29	0.33	0.05	116	9748.07	39.36	0.26	0.04	95	9771.3
	2.8	39.29	0.33	0.05	116	9748.40	39.37	0.26	0.04	95	9771.59
t_d	0.0493	39.30	0.32	0.05	116	9759.06	39.38	0.26	0.04	94	9782.51
	0.0657	39.30	0.32	0.05	116	9753.24	39.37	0.26	0.04	95	9776.60
	0.0986	39.28	0.33	0.05	117	9742.61	39.36	0.27	0.04	95	9765.64
	0.1151	39.27	0.33	0.05	117	9737.76	39.35	0.27	0.04	95	9760.60
b_1	48	39.15	0.31	0.08	120	9694.49	39.23	0.25	0.07	98	9716.95
	64	39.23	0.32	0.06	118	9726.87	39.31	0.26	0.05	96	9749.76
	96	39.33	0.33	0.04	115	9762.36	39.40	0.27	0.04	94	9785.76
	112	39.36	0.33	0.04	115	9773.14	39.43	0.27	0.03	93	9796.72
l_1	36	39.28	0.33	0.05	117	9746.44	39.36	0.26	0.04	95	9769.65
	48	39.29	0.33	0.05	117	9747.11	39.36	0.26	0.04	95	9770.32
	72	39.29	0.33	0.05	116	9748.37	39.37	0.26	0.04	95	9771.61
	84	39.29	0.33	0.05	116	9749.04	39.37	0.26	0.04	95	9772.25
θ	0.024	39.26	0.34	0.05	120	9731.19	39.34	0.27	0.04	97	9753.99
	0.032	39.28	0.33	0.05	118	9731.34	39.35	0.27	0.04	96	9762.58
	0.048	39.30	0.32	0.05	115	9755.67	39.38	0.26	0.04	94	9779.09
	0.056	39.30	0.32	0.05	115	9763.48	39.39	0.26	0.04	93	9787.01
d	8	35.17	0.31	0.05	112	7959.25	35.24	0.25	0.04	88	7987.49
	10	39.29	0.33	0.05	116	9747.76	39.36	0.26	0.04	95	9770.96
	12	43.01	0.34	0.06	121	11533.36	43.09	0.28	0.05	101	11552.92
	14	46.44	0.34	0.06	125	13316.44	46.52	0.29	0.05	106	13333.29
F_{up}	1.2	39.36	0.33	0.05	117	8268.10	39.43	0.27	0.04	96	8291.12
	1.6	39.32	0.33	0.05	117	9007.97	39.40	0.27	0.04	95	9031.1
	2.4	39.25	0.32	0.05	116	10487.43	39.33	0.26	0.04	94	10510.74
	2.8	39.21	0.32	0.05	116	11227.01	39.30	0.26	0.05	94	11250.42
δ	0.03	39.29	0.3256	0.05	117	9747.11	39.36	0.2641	0.04	95	9770.34
	0.04	39.29	0.3257	0.05	117	9747.47	39.36	0.2641	0.04	95	9770.62
	0.06	39.29	0.3258	0.05	116	9748.09	39.36	0.2642	0.04	95	9771.28
	0.07	39.29	0.3259	0.05	116	9748.38	39.36	0.2643	0.04	95	9771.61

➤ **Parameters variation effects on supply chain total cost**

- It has been observed that the demand parameters α , β and γ increases then total cost also increases.
- Total cost will be decreases due to the increases the value of ξ , η and t_d in both VMI model as well as traditional model.
- Ordering cost of supplier A_s and buyer ordering cost A_b proportional to the total cost. In actuality, as buyer ordering costs increase, overall inventory cost after VMI increases as well. Thus, the ordering cost of the buyer is very sensitive. However, the use of VMI reduces the overall costs of inventory compared with traditional supply chains. Carbon emission parameters e_h , e_d , e_{T1} and e_{T2} increases then supply chain total cost also increases.

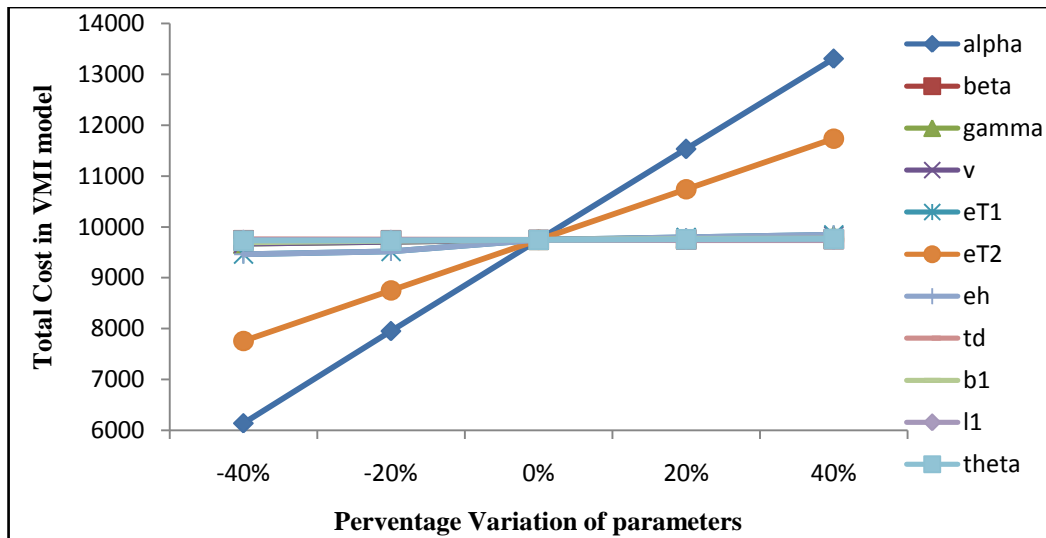


Figure 7.8 Effect of inventory parameters on total cost in VMI model

- If the increases the C_{fT} , C_{VT} , F_{ue} , F_{up} and increases the distance d then total cost of supply chain also increases in traditional model as well VMI model.

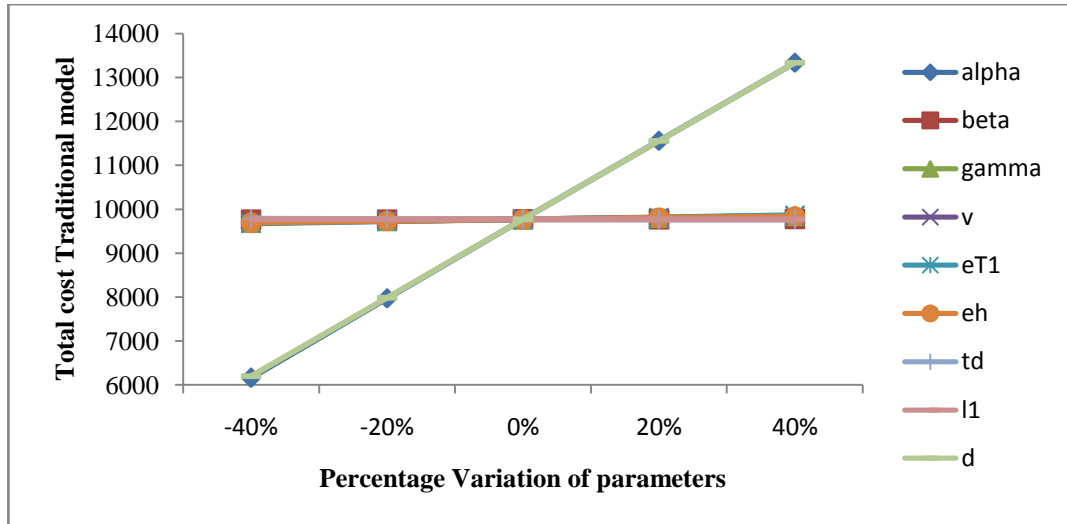


Figure 7.9 Effect of inventory key parameters on total cost in traditional model

- The promotional level v increases the demand is also increase but total supply chain cost slightly increases.
- When δ increases, there is an increase in total inventory cost.
- The total cost of supply chain is lower in vendor managed inventory system compare to traditional supply chain inventory system for all parameters.
- **Parameters variation effects on green investment cost and carbon emission cost**
 - Notice that the increases the parameters η then green investment cost and total supply chain cost will be increases with carbon emission cost decreases, but the higher value of ξ decreases the carbon emission cost with increases the green investment.
 - Increases the constant market demand then total cost, emission cost increases with increasing green investment cost. Here observed that β increases then total cost and carbon emission cost increases with decreases green investment.
 - Promotional level v and constant coefficient γ increases then green investment also increases in VMI system as well as traditional model for shortages and without shortages cases.
 - Carbon emission parameters e_{T1}, e_{T2}, e_h and e_d are proportional to the green investment cost and total cost, with negative proportional to the carbon emission cost.
 - Higher non deterioration period t_d resulted to lower green investment cost and lower total supply chain cost, with rising to carbon emission cost.

- If the increases A_s and A_b then slightly decreases green investment cost with decreases carbon emission cost but increases the supply chain total cost in VMI model and traditional model both.
 - If the increases in F_{ue} , F_{up} then the total cost highly increases with decreases the order quantity and cycle time.
- **Parameters variation effect on replenishment cycle time and ordering quantity**
- If increases in α then cycle length and ordering quantity increases highly. On changing in parameter β, γ , there is no change in cycle time and slightly varies ordering quantity. If the increases the promotional level v then positive cycle time decreases in both model, but shortages period t_2 and t_{2V} increases.
 - The ordering quantity decreases due to increases v in VMI. If e_{T1} increases then Q_V^* , Q^* and t_1, t_{1V}, t_2, t_{2V} increases but when e_{T2}, e_h, e_d increases then Q_V^* , Q^* and t_1, t_{1V}, t_2, t_{2V} decreases.
 - Ordering cost of buyer and supplier is proportional to the ordering quantity and cycle time but holding cost and deterioration cost are negative proportional to the ordering quantity and cycle time.
 - If the parameters b_1, l_1 , θ increases then ordering quantity decreases.
 - When δ increases, there is cycle length slightly increases and order quantity decreases. If the increases in F_{up} , F_{ue} cycle time also increases slightly. Other remaining parameters variation not much effect on cycle time and ordering quantity.

7.6 Discussion about managerial insights

From the above numerical and sensitivity analysis, following insights can be derived:

- Investment on green technology useful to reduces the total carbon emission, and hence reduced the total carbon emission cost. Carbon emission cost without green investment and carbon emission cost with green investment has vast different, it is recommended that the decision maker should investment in green technology is necessary to reduces the major effect of carbon emission and their cost.

- Role of promotional level is positive with respect to demand and green investment, higher promotinal level of green investment increases the green investment cost, higher green investment reduces carbon emission.
- Distance between the supplier and retailer warehouse should be minimum, due to the short distance may diminished utility of fuel and carbon emission. The total cost increases faster as the distance gets higher because the related expenditure also develops significantly. This study shows that fuel utilization directly impact on cost, and total cost and emission increases highly. It is indicated that the firm's may adopt the electric vehicle as a green initiative to minimize the cost as well as environmental effects.
- In order to make it simple for the management to choose the products, the model anticipates non-instantaneous deteriorating products. Our analysis indicated that the higher duration of non-deteriorating is help to reduces the cost and carbon emission.
- This model demonstrates the precise parameters and period the shortage starts, together with whether it is favourable to the manager.

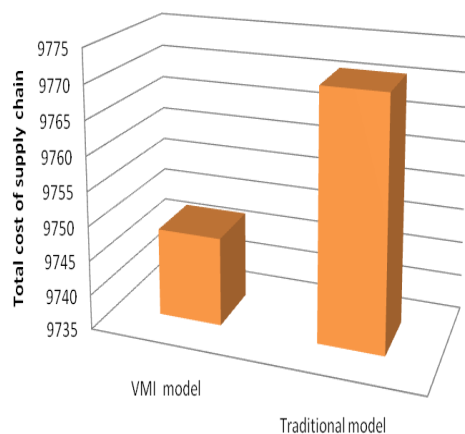


Figure 7.10 Total cost of supply chain in both models

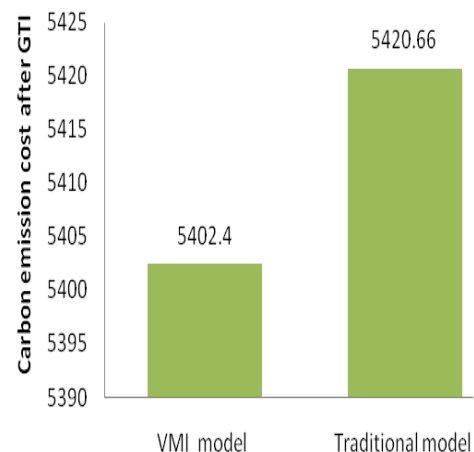


Figure 7.11 Carbon emission cost after GTI in both models

- From analysis shows that the vendor managed inventory model with partially backlog shortages model is best because the supply chain cost is minimum with carbon emission cost also minimum.
- From above analysis, it is suggested that decision maker should be apply VMI model for minimize the total cost of supply chain and maximum reduces to carbon emission and carbon emission cost.

- Decision maker need to focus more on the efficiency of carbon reduction, the emission factor of green technology, and the carbon emission parameters from transportation logistics and storage when it comes to the overall decision model because variations to these parameters have a big impact on the total cost.
- Higher backloging rate increases to total cost, a decision maker should maintain the backloging rate, back order cost and lost sale cost for minimize the total cost.
- Investment in green technology in VMI model and traditional reduces the total cost of supply chain. (Table 7.3)

7.7 Conclusion

In this chapter, we investigates the optimum values of the green investment cost, replenishment cycle time and ordering quantity such that the supply chain total cost and carbon emission cost is minimized in Vendor managed inventory system as well as traditional supply chain model. Ultimately, developed two models i.e. traditional model and VMI model. VMI system and individual system is developed for single vendor and single buyer for green investment and its promotional level dependent demand and permitting shortages with partial backloging with non instantaneous deterioration product. The convexity of total cost function proved by theoretically and graphically. Sensitivity analysis is exhibited to show the liability of the model. It is noticed that total average cost is decreases in VMI system compare to system developed by individual effort of vendor and buyer. This chapter filled the research gaps as the considering green investments policy, demand which depends on green investments as a rational function of carbon emission reduction and promotion of green investment in VMI system and traditional inventory system. Another important aspect of this chapter is that the transportation cost, which depends on the number of trips of the vehicle, product weight, and fuel utilization, is considered, and emissions from transportation also depend on the distance between two players. Emissions from holding inventory and from deterioration are also considered. How to impact the different costs and emissions on the overall supply chain cost is determined. Total cost is lower in VMI system compared to tradition system, green investments directly reduces the emissions, promotion of green investment increases demand, higher non deterioration period help to minimize the total cost, emission level and total cost increases with increases in fuel utility, long duration of shortages period and higher backloging rate will be negative impact on business, are the key outcomes of our

study. Some of the limitations of proposed chapter are, in inventory management single supplier and single buyer is not always possible, a big company have a single vendor and multiple buyers, Three sources of carbon emission are considered this proposed model but in real situation many sources of carbon emissions, there for we may develop more precise green technology for reducing carbon emission, carbon trading mechanism not mention here, a firm may apply the carbon trading mechanism as per government policy as a further study, the market demand of product depends on many criteria's. This model will be extended as a future scope of research with considering time dependent deterioration rate, single vendor-multiple buyer concept, to apply carbon tax or cap and trade mechanism to minimize the total cost and protect environment from harmful carbon emission etc.

CHAPTER-8

Optimal Greening Efforts, Pricing and Inventory Strategies for Non Instantaneous Deteriorating Perishable Products under Price, Freshness and Green Efforts Dependent Demand with Price Discount

8.1 Introduction

Customers who feel anxious about their health these days look for and anticipate wholesome, environmentally friendly goods. The utilization of sustainable fresh perishable products has gained popularity among individuals due to their attributes of freshness, healthfulness, and environmental friendliness. The pricing of products significantly influences consumers purchasing behaviours. This chapter looks to extend the work of Chen et al.[75], Raza and Faisal[229] to formulate the EOQ model for non-instantaneous deteriorating products with freshness, selling price, and greening efforts demand. The previous literatures emphasize several types of methodologies that have been published in the area of green inventory management. Two crucial components of market demand are the product's freshness and pricing. After reviewing the published literature, none of the researchers considered the idea of coupling greening initiatives with demand related to freshness and pricing. Demand for perishable products such as organic agriculture products, packaged beverage products, and dairy products, were determined not only by freshness and price, but also by the consumer's preference for greenness. Other novel idea is, at the beginning of their shelf lives,

perishable products may not deteriorate physically as well as qualitatively more quickly, and their freshness does not degrade as noticeably. Hence, demand during the beginning of inventory cycle time is price and green efforts dependent taken. After some period, products affected by deterioration of both type and product value degrade continuously, during this period demand pattern is taken depend on freshness, price and greening efforts; and the retailer offers a price discount during the deterioration-affected period for boosting demand. The study looks for to maximize the retailer's total profit by taking all factors into account. The objectives of this chapter are to establish the optimum selling price, the optimum replenishment cycle time, and the optimal cost of greening efforts while also focusing on the retailer's total profit maximization. Numerical results are used to develop and validate a mathematical model that reflects real-world circumstances. Optimality at the decisions variables of the objective verified through theoretical results and graphically. Sensitivity analysis for the parameters is done to evaluate the model's stability. The chapter concludes with a discussion of the potential future direction of the associated study after presenting key managerial implications. The management insights and conclusion section provides a summary of the study's challenges and other noteworthy findings.

8.2 Notations and Assumptions

The following notations, assumptions are used to develop the mathematical structure in this chapter:

8.2.1 Notations

Parameters

A	Ordering cost (in ₹/order)
h	Constant holding cost (₹/unit/unit time).
C	Purchase cost per item (constant) (₹/unit).
t_d	Non deterioration period (year).
τ	Maximum life-time of the product beyond which no consumer will buy the product. $0 < t_d < T \leq \tau$.
p_0	Rate of discount (in percentage) offered by retailer from t_d on selling price.
λ	Greening efforts effectiveness parameter ($\lambda > 0$)

θ Constant rate of deterioration of product, $0 < \theta < 1$.

Decision variables

T Cycle time; (in years).

g_e Cost of greening efforts; (in ₹/unit).

p Selling Price (in ₹/unit), $p > C$.

Objective function

$TP(p, T, g_e)$ Retailer's/firm's total profit per cycle (in ₹).

Expressions and functions

$R(p, t, g_e)$ Demand function at time t .

$I_1(t)$ Inventory level at time t during the time interval $[0, t_d]$.

$I_2(t)$ Inventory level at time t during the time interval $[t_d, T]$.

Q Ordering quantity (units).

8.2.2 Assumptions

1. There is just one kind of perishable product for which the inventory system is appropriate.
2. The deterioration rate of product is non instantaneous constant. i.e. inventory time $t = 0$ to $t = t_d$ there is no effect of physical deterioration and product is fully fresh but after time $t = t_d$ to $t = T$ product affect by two different kinds of deterioration over time: a physical deterioration at a constant rate θ ($0 < \theta < 1$) of the existing inventory, and a degradation of the product's freshness quality.
3. Deteriorated product cannot be repaired or replaced and products have no salvage value.
4. Retailer/firm offered a discount p_0 in selling price to the customers during deteriorating period, i.e. $0 \leq t \leq t_d$.
5. A number of elements such as time spent on resources, rate of warmth, temperature, and preservation, among others, may have an influence on the product's freshness. It appears to be impossible to obtain a product's explicit freshness level. Nevertheless, it goes without mentioning that any product's freshness gradually degrades and wears out over time. Therefore, we can assume that the freshness index is 1 at time 0 and gradually decreases over time until it eventually reaches 0 as the product gets closer to

- its expire (i.e, it cannot be sold). The freshness index is $f(t) = \frac{\tau - t}{\tau}, 0 \leq t \leq \tau$. (Chen et al[75], Dobson[211], Agi and Soni[212], Soni[215]).
6. The replenishment cycle time is shorter than the longest possible product shelf life. i.e. $T \leq \tau$ and inventory level reaches zero at T , means there is no inventory remaining at time $t = T$.
 7. The firm's/retailer's greening effort requires a capital investment in greening efforts over a certain time frame rather than raising the unit price of the product. The firm/retailer uses green investments to produced or maintain organic/ green products.
 8. Total greening investments per unit time t is $\int_0^t (\lambda \cdot g_e) dg_e dt = \frac{\lambda \cdot g_e^2 \cdot t}{2}$. So, the total investment for the time duration T is $\frac{\lambda \cdot g_e^2 \cdot T}{2}$. (Swami and Shah[227], and Ghosh and Shah[198], Raza and Faisal[229], Shah et al.[232]).
 9. The demand rate is based on selling price of product, freshness index, i.e. age of product as well as green efforts. Demand rate function mathematically defined as,
$$R(p, t, g_e) = \begin{cases} r(p) + \gamma g_e, & 0 \leq t \leq t_d \\ r(p_d) f(t - t_d) + \gamma g_e, & t_d \leq t \leq T \end{cases}$$
, in this expression $r(p) = \alpha - \beta p$ and $r(p_d) = \alpha - \beta p(1 - p_0)$, are any non-negative, continuous, convex decreasing functions of selling price, and $\alpha > 0$ represent the constant market demand, $\beta > 0$ is the price elasticity factor, $\gamma > 0$ is greening investments effectiveness scale.
 10. The lead time is negligible and the replenishment rate infinite and shortages are not permissible.

8.3 Mathematical formulation

The evolution of the inventory framework is as follows: Consequently, at the start of each cycle, Q units of fresh perishable products are stored in the inventory system. In other words, at time $t = 0$, Q units of the fresh green perishable products in the stock in inventory system and there no deterioration effect and product to be considered fully fresh i.e $f(t) \rightarrow 1, 0 \leq t \leq t_d$ and the inventory level drops due to demand only during $0 \leq t \leq t_d$.

The position of inventory level of the products at time t over the period $[0, t_d]$ is governed by the following differential equation,

$$\frac{dI_1(t)}{dt} = -(r(p) + \gamma g_e), 0 \leq t \leq t_d \quad (8.1)$$

Inventory levels diminish over time span $0 \leq t \leq t_d$ as a result of the combined effects of demand, physical deterioration, and a reduction in the freshness degree of the products, and inventory levels attain zero at the end of cycle time $t = T$, the retailer offered a price discount p_0 percentage on original selling price during $0 \leq t \leq t_d$. The condition of inventory level of the product at time t over the $[0, t_d]$ is governed by the following differential equation,

$$\frac{dI_2(t)}{dt} + \theta I_2(t) = -(r(p_d)f(t - t_d) + \gamma g_e), t_d \leq t \leq T \leq \tau \quad (8.2)$$

The solution of (8.1) at the boundary condition $I_1(0) = Q$ and solution of (8.2) at $I_2(T) = 0$ is respectively,

$$I_1(t) = Q - (r(p) + \gamma g_e)t \quad (8.3)$$

$$I_2(t) = \left((e^{\theta(T-t)} - 1) \left(\frac{\gamma g_e}{\theta} + \frac{r(p_d)}{\theta} + \frac{r(p_d)t_d}{\tau\theta} + \frac{r(p_d)}{\tau\theta^2} \right) - \frac{r(p_d)}{\tau\theta} (e^{\theta(T-t)}T - t) \right) \quad (8.4)$$

Since, $I_1(t)$ and $I_2(t)$ are continues function at $t = t_d$, $I_1(t_d) = I_2(t_d)$, that gives,

$$Q = (r(p) + \gamma g_e)t_d + \left((e^{\theta(T-t_d)} - 1) \left(\frac{\gamma g_e}{\theta} + \frac{r(p_d)}{\theta} + \frac{r(p_d)t_d}{\tau\theta} + \frac{r(p_d)}{\tau\theta^2} \right) - \frac{r(p_d)}{\tau\theta} (e^{\theta(T-t_d)}T - t_d) \right) \quad (8.5)$$

From (8.3) and (8.5), we have

$$I_1(t) = (r(p) + \gamma g_e)(t_d - t) + \left((e^{\theta(T-t_d)} - 1) \left(\frac{\gamma g_e}{\theta} + \frac{r(p_d)}{\theta} + \frac{r(p_d)t_d}{\tau\theta} + \frac{r(p_d)}{\tau\theta^2} \right) - \frac{r(p_d)}{\tau\theta} (e^{\theta(T-t_d)}T - t_d) \right) \quad (8.6)$$

Now, we'll go over several model-related costs in the following manner:

Ordering costs are expenses incurred when a product is ordered.

$$\text{Cost of ordering per order: } OC = A \quad (8.7)$$

Cost of holding per unit per unit time:

$$\begin{aligned}
 HC &= h \left[\int_0^{t_d} I_1(t) dt + \int_{t_d}^T I_2(t) dt \right] \\
 &= h \left[\frac{t_d^2}{2} (r(p) + \gamma g_e) + \left((e^{\theta(T-t_d)} - 1) \left(\frac{\gamma g_e}{\theta} + \frac{r(p_d)}{\theta} + \frac{r(p_d)t_d}{\tau\theta} + \frac{r(p_d)}{\tau\theta^2} \right) - \frac{r(p_d)}{\tau\theta} (e^{\theta(T-t_d)} T - t_d) \right) t_d \right. \\
 &\quad \left. + \left(\left(-\frac{e^{\theta t_d}}{\theta} - (T - t_d) \right) \left(\frac{\gamma g_e}{\theta} + \frac{r(p_d)}{\theta} + \frac{r(p_d)t_d}{\tau\theta} + \frac{r(p_d)}{\tau\theta^2} \right) + \frac{r(p_d)}{\tau\theta} \left(\frac{T e^{\theta t_d}}{\theta} + \frac{T^2 - t_d^2}{2} \right) \right) \right]
 \end{aligned} \tag{8.8}$$

Purchase cost: $PC = CQ$

$$= C \left[(r(p) + \gamma g_e) t_d + \left((e^{\theta(T-t_d)} - 1) \left(\frac{\gamma g_e}{\theta} + \frac{r(p_d)}{\theta} + \frac{r(p_d)t_d}{\tau\theta} + \frac{r(p_d)}{\tau\theta^2} \right) - \frac{r(p_d)}{\tau\theta} (e^{\theta(T-t_d)} T - t_d) \right) \right] \tag{8.9}$$

Deteriorating cost:

$$\begin{aligned}
 DC &= C_d \left[I_2(t_d) - \int_{t_d}^T r(p_d) \left(1 - \frac{t-t_d}{\tau} \right) + \gamma g_e \right] dt \\
 &= C_d \left[\left((e^{\theta(T-t_d)} - 1) \left(\frac{\gamma g_e}{\theta} + \frac{r(p_d)}{\theta} + \frac{r(p_d)t_d}{\tau\theta} + \frac{r(p_d)}{\tau\theta^2} \right) - \frac{r(p_d)}{\tau\theta} (e^{\theta(T-t_d)} T - t_d) \right) \right. \\
 &\quad \left. - \left((T - t_d) \left(r(p_d) \left(1 - \frac{T+t_d}{2\tau} + \frac{t_d}{\tau} \right) + \gamma g_e \right) \right) \right]
 \end{aligned} \tag{8.10}$$

Depending on the product's level of greening, retailers may have to incur extra expenses to increase the product's quality. Greening efforts investment is,

$$GEI = \int_0^T \int_0^{g_e} \lambda \cdot g_e dg_e dt = \frac{\lambda \cdot g_e^2 \cdot T}{2} \tag{8.11}$$

The revenue generated by each sales cycle is,

$$\begin{aligned}
 SR &= p \int_0^{t_d} r(p) + \gamma g_e dt + p(1 - p_0) \int_{t_d}^T r(p_d) \left(1 - \frac{t-t_d}{\tau} \right) + \gamma g_e dt \\
 &= p(r(p) + \gamma g_e) t_d + p(1 - p_0) (T - t_d) \left(r(p_d) \left(1 - \frac{T+t_d}{2\tau} + \frac{t_d}{\tau} \right) + \gamma g_e \right)
 \end{aligned} \tag{8.12}$$

Retailer's total profit per cycle time T is formulated as,

$$TP(T, g_e, p) = \frac{1}{T} (SR - OC - PC - HC - DC - GEI) \quad (8.13)$$

$$= \frac{1}{T} \left[\begin{aligned} & p(r(p) + \gamma g_e) t_d + p(1 - p_0)(T - t_d) \left(r(p_d) \left(1 - \frac{T + t_d}{2\tau} + \frac{t_d}{\tau} \right) + \gamma g_e \right) \\ & - A - h \left[\begin{aligned} & \frac{t_d^2}{2} (r(p) + \gamma g_e) + \left((e^{\theta(T-t_d)} - 1) \left(\frac{\gamma g_e}{\theta} + \frac{r(p_d)}{\theta} + \frac{r(p_d)t_d}{\tau\theta} + \frac{r(p_d)}{\tau\theta^2} \right) \right) t_d \\ & - \frac{r(p_d)}{\tau\theta} (e^{\theta(T-t_d)} T - t_d) \end{aligned} \right] \\ & + \left(\left(-\frac{e^{\theta t_d}}{\theta} - (T - t_d) \right) \left(\frac{\gamma g_e}{\theta} + \frac{r(p_d)}{\theta} + \frac{r(p_d)t_d}{\tau\theta} + \frac{r(p_d)}{\tau\theta^2} \right) \right) \\ & + \frac{r(p_d)}{\tau\theta} \left(\frac{T e^{\theta t_d}}{\theta} + \frac{T^2 - t_d^2}{2} \right) \end{aligned} \right] \\ & - C \left[\begin{aligned} & (r(p) + \gamma g_e) t_d + \left((e^{\theta(T-t_d)} - 1) \left(\frac{\gamma g_e}{\theta} + \frac{r(p_d)}{\theta} + \frac{r(p_d)t_d}{\tau\theta} + \frac{r(p_d)}{\tau\theta^2} \right) \right) \\ & - \frac{r(p_d)}{\tau\theta} (e^{\theta(T-t_d)} T - t_d) \end{aligned} \right] \\ & - C_d \left[\begin{aligned} & \left((e^{\theta(T-t_d)} - 1) \left(\frac{\gamma g_e}{\theta} + \frac{r(p_d)}{\theta} + \frac{r(p_d)t_d}{\tau\theta} + \frac{r(p_d)}{\tau\theta^2} \right) - \frac{r(p_d)}{\tau\theta} (e^{\theta(T-t_d)} T - t_d) \right) \\ & - \left((T - t_d) \left(r(p_d) \left(1 - \frac{T + t_d}{2\tau} + \frac{t_d}{\tau} \right) + \gamma g_e \right) \right) \end{aligned} \right] - \frac{\lambda \cdot g_e^2 \cdot T}{2} \end{aligned} \right]$$

8.3.1 Solution technique to determine the optimal solution

In this sub-section, we derive the optimal value of decision variables T, g, p which maximize the total profit per unit time of the inventory system. To prove this result the optimal solution must satisfy the necessary condition,

$$\frac{\partial TP}{\partial T} = 0, \quad \frac{\partial TP}{\partial g_e} = 0 \text{ and } \frac{\partial TP}{\partial p} = 0$$

Taking the first order partial derivative of (8.13), with respect to T and equating to zero, we have,

$$\frac{\partial TP}{\partial T} = \left[\begin{array}{l} \frac{-1}{T^2} (p(r(p) + \gamma g_e) t_d) + p(1 - p_0) (\Phi_2) + \frac{A}{T^2} \\ -h \left[\begin{array}{l} -\frac{t_d^2}{2T^2} (r(p) + \gamma g_e) + \left(\left(\frac{e^{\theta(T-t_d)} (T\theta - 1)}{T^2} + \frac{1}{T^2} \right) \Phi_1 \right) t_d \\ -\frac{r(p_d)}{\tau\theta} (\theta e^{\theta(T-t_d)} + \frac{t_d}{T^2}) \end{array} \right] \\ + \left(\left(\frac{e^{\theta t_d}}{T^2\theta} - \frac{t_d}{T^2} \right) \Phi_1 + \frac{r(p_d)}{\tau\theta} \left(\frac{1}{2} + \frac{t_d^2}{2T^2} \right) \right) \\ -C \left[\begin{array}{l} -\frac{1}{T^2} (r(p) + \gamma g_e) t_d + \left(\frac{e^{\theta(T-t_d)} (T\theta - 1)}{T^2} + \frac{1}{T^2} \right) \Phi_1 \\ -\frac{r(p_d)}{\tau\theta} (\theta e^{\theta(T-t_d)} + \frac{t_d}{T^2}) \end{array} \right] \\ -C_d \left[\left(\frac{e^{\theta(T-t_d)} (T\theta - 1)}{T^2} + \frac{1}{T^2} \right) \Phi_1 - \frac{r(p_d)}{\tau\theta} (\theta e^{\theta(T-t_d)} + \frac{t_d}{T^2}) \right] - (\Phi_2) \end{array} \right] = 0 \quad (8.14)$$

where, $\Phi_1 = \left(\frac{\gamma g_e}{\theta} + \frac{r(p_d)}{\theta} + \frac{r(p_d) t_d}{\tau\theta} + \frac{r(p_d)}{\tau\theta^2} \right)$ and

$$\Phi_2 = \left(r(p_d) \left(\frac{t_d}{T^2} - \frac{1}{2\tau} - \frac{t_d^2}{2\tau T^2} + \frac{t_d^2}{T^2\tau} \right) + \gamma g_e \frac{t_d}{T^2} \right)$$

Taking the first order partial derivative of (9.13), with respect to g_e and equating to zero,

$$\frac{\partial TP}{\partial g_e} = \frac{1}{T} \left[\begin{array}{l} p\gamma t_d + p(1 - p_0)(T - t_d)(\gamma) \\ -h \left(\frac{\gamma t_d^2}{2} + \frac{\gamma}{\theta} \left((e^{\theta(T-t_d)} - 1) t_d - \frac{e^{\theta t_d}}{\theta} - (T - t_d) \right) \right) \\ -C \left(\frac{\gamma}{\theta} (e^{\theta(T-t_d)} - 1) \right) - C_d \frac{\gamma}{\theta} (e^{\theta(T-t_d)} - 1) - \gamma(T - t_d) - \lambda \cdot g_e \cdot T \end{array} \right] = 0 \quad (8.15)$$

Taking the first order partial derivative of (9.13), with respect to p and equating to zero,

$$\frac{\partial TP}{\partial p} = \frac{1}{T} \left[\begin{array}{l} (\alpha - 2\beta p + \gamma g_e)t_d + (1 - p_0)(T - t_d)((\alpha - 2\beta p + 2\beta p p_0)\Phi_5 + \gamma g_e) \\ -h \left[\frac{-\beta t_d^2}{2} + ((e^{\theta(T-t_d)} - 1)\Phi_3 - \Phi_4)t_d + \left(\left(-\frac{e^{\theta t_d}}{\theta} - (T - t_d) \right) \Phi_3 \right) \right. \\ \left. + \frac{(-\beta + \beta p_0)}{\tau\theta} \Phi_6 \right] \\ -C \left[(-\beta)t_d + ((e^{\theta(T-t_d)} - 1)\Phi_3 - \Phi_4) \right] \\ -C_d \left[((e^{\theta(T-t_d)} - 1)\Phi_3 - \Phi_4) - ((T - t_d)((-\beta + \beta p_0)\Phi_5)) \right] \end{array} \right] = 0 \quad (8.16)$$

where, $\Phi_3 = (-\beta + \beta p_0) \left(\frac{1}{\theta} + \frac{t_d}{\tau\theta} + \frac{1}{\tau\theta^2} \right)$, $\Phi_4 = \frac{(-\beta + \beta p_0)}{\tau\theta} (e^{\theta(T-t_d)}T - t_d)$,

$$\Phi_5 = \left(1 - \frac{T + t_d}{2\tau} + \frac{t_d}{\tau} \right), \quad \Phi_6 = \left(\frac{T e^{\theta t_d}}{\theta} + \frac{T^2 - t_d^2}{2} \right)$$

Theorem 8.1: For given positive p and g_e , if the total profit of inventory system is a strictly concave function of T , then

(a) Equation (8.14) have a one and only one solution.

(b) The solution in (a) satisfied the second order condition for the maximum.

Proof: Let's take the any positive fix value p of and g_e , taking the partial derivative of (8.14) with respect to T , we have

$$\frac{\partial^2 TP}{\partial T^2} = \left[\begin{array}{l} \frac{2}{T^3} (p(r(p) + \gamma g_e)t_d) - p(1 - p_0)\varphi_1 - \frac{2A}{T^3} \\ -h \left[\frac{t_d^2}{T^3} (r(p) + \gamma g_e) + (\varphi_2\Phi_1 - \varphi_3)t_d + \left(\varphi_4\Phi_1 - \frac{r(p)}{\tau\theta} \frac{t_d^2}{T^3} \right) \right] \\ -C \left[\frac{2}{T^3} (r(p) + \gamma g_e)t_d + (\varphi_2\Phi_1 - \varphi_3) \right] - C_d \left[(\varphi_2\Phi_1 - \varphi_3) - \varphi_1 \right] \end{array} \right] \quad (8.17)$$

In above expression, $\varphi_1 = \left(-r(p_d) \left(\frac{2t_d}{T^3} + \frac{t_d^2}{\tau T^3} \right) - \gamma g_e \frac{2t_d}{T^3} \right)$,

$$\varphi_2 = \left(\frac{e^{\theta(T-t_d)}(T^2\theta^2 - T\theta + 2)}{T^3} - \frac{2}{T^3} \right), \quad \Phi_1 = \left(\frac{\gamma g_e}{\theta} + \frac{r(p_d)}{\theta} + \frac{r(p_d)t_d}{\tau\theta} + \frac{r(p_d)}{\tau\theta^2} \right),$$

$$\varphi_3 = \frac{r(p_d)}{\tau\theta} (\theta^2 e^{\theta(T-t_d)} - \frac{t_d}{T^3}) \text{ and } \varphi_4 = \left(\frac{2t_d}{T^3} - \frac{2e^{\theta t_d}}{T^3\theta} \right) \text{ we can notice that Noticed that,}$$

$$\frac{2}{T^3} (p(r(p) + \gamma g_e) t_d) < p(1-p_0)\varphi_1, \varphi_2\Phi_1 > \varphi_3, \varphi_4\Phi_1 > \frac{r(p_d)}{\tau\theta} \frac{t_d^2}{T^3}.$$

It is concluded that, $\frac{\partial^2 TP}{\partial T^2} < 0$ at T^* for the positive fix value of p and g_e . (8.18)

Hence, (8.14) has a unique solution and satisfied the sufficient condition for maxima. It is concluded that for given positive fix value of p and g_e the solution T^* which maximize (8.13) not only exists but is also unique.

Theorem 8.2: *For given value of p and T , if the total profit of inventory system is a strictly concave function of g_e , then*

(a) *Equation (8.15) have a one and only one solution.*

(b) *The solution in (a) satisfied the second order condition for the maximum.*

Proof: Let's take the any positive fix value of p and T , taking partial derivative of (8.15) with respect to g_e , we have

$$\frac{\partial^2 TP}{\partial g_e^2} = -\lambda < 0 \text{ at } (T^*, g_e^*). \quad (8.19)$$

Therefore, there exist a unique value of g_e which maximize (8.13).

Theorem 8.3: *For any given positive selling price p , total profit function satisfied the Hessian matrix conditions for concavity at the optimal value of T^* and g_e^* .*

Proof: The partial derivative of (8.15) with respect to T and simplifying terms is given as follows:

$$\frac{\partial^2 TP}{\partial g_e \partial T} = \begin{bmatrix} -\frac{p\gamma t_d}{T^2} + p(1-p_0)\left(\frac{t_d}{T^2}\right)(\gamma) \\ -h\left(-\frac{\gamma t_d^2}{2T^2} + \frac{\gamma}{\theta}\left(\frac{e^{\theta(T-t_d)}(T\theta-1)}{T^2} + \frac{1}{T^2}\right)t_d + \frac{e^{\theta t_d}}{T^2\theta} - \frac{t_d}{T^2}\right) \\ -C\left(\frac{\gamma}{\theta}\left(\frac{e^{\theta(T-t_d)}(T\theta-1)}{T^2} + \frac{1}{T^2}\right)\right) - C_d \frac{\gamma}{\theta}\left(\frac{e^{\theta(T-t_d)}(T\theta-1)}{T^2} + \frac{1}{T^2}\right) - \gamma \frac{t_d}{T^2} \end{bmatrix} \quad (8.20)$$

From (9.18), (9.19) and (9.20), observed that

$$\left[\frac{\partial^2 TP}{\partial T^2} \right]_{(T^*, g_e^*)} \geq \left[\frac{\partial^2 TP}{\partial g_e \partial T} \right]_{(T^*, g_e^*)} \quad \text{and} \quad \left[\frac{\partial^2 TP}{\partial g_e^2} \right]_{(T^*, g_e^*)} \geq \left[\frac{\partial^2 TP}{\partial g_e \partial T} \right]_{(T^*, g_e^*)}$$

Using the above conditions, the value of the Hessian matrix at (T^*, g_e^*) is given by,

$$|H| = \left[\left[\frac{\partial^2 TP}{\partial T^2} \right] \left[\frac{\partial^2 TP}{\partial g_e^2} \right] - \left[\frac{\partial^2 TC}{\partial g_e \partial T} \right]^2 \right]_{(T^*, g_e^*)} > 0 \quad (8.21)$$

then the hessian matrix associated with $TP(T, g_e, p)$ is negative definite. Therefore, total profit function $TP(T, g_e, p)$ in (8.13) is concave function in T^* and g_e^* for the fix positive value of p .

Theorem 8.4: *For the optimum value of T^* and g_e^* , if the total profit is a strictly concave function of p , then*

(a) *There exists a unique optimal p^* that satisfies (8.16).*

(b) *The solution of (a) satisfied the second order condition for the maxima.*

Proof: Taking the second order partial derivative of the (8.13) with respect to p , with given value of T^* and g_e^* ,

$$\frac{\partial^2 TP}{\partial p^2} = \frac{1}{T^*} \left[-2\beta t_d - 2\beta(1-p_0)^2(T^* - t_d) \left(\frac{2\tau - T^* + t_d}{2\tau} \right) \right]$$

$$\text{Since, } \tau \geq T^* \geq t_d, \text{ so } \frac{2\tau - T^* + t_d}{2\tau} > 0, \text{ Hence, } \frac{\partial^2 TP}{\partial p^2} < 0 \text{ at } p^*. \quad (8.22)$$

Hence the equation (8.16) has a unique solution and satisfies the second order condition for the maximum. Hence, for given positive fix value of T^* and g_e^* the solution p^* which maximize (8.13) not only exists but is also unique. The conditions (8.21) and (8.22) also proved by numerically next section. Proof of theorem 4 completed.

The following solutions procedure adopted to optimize the value of decision variables and objective function by using the mathematical software like maple 18 or matlab or mathematica.

Step 1: First, give the inventory parameters any specific speculative values.

Step 2: Choose any positive fix value of p ($p > C$).

Step 3: Solving the simultaneous equations stated in (8.14) and (8.15) using the mathematical software like maple XVIII or Matlab, to find T^* and g_e^* .

Step 4: Verify the sufficient conditions stated in (8.18), (8.19) and (8.21) at T^* and g_e^* , if not satisfied go to step 2 and choose other value of p and other value of parameters in step 1, repeat process till (8.18), (8.19) and (8.21) satisfied.

Step 5: Solve (8.15), to find p^* at T^* and g_e^* , verify (8.22) other go to step 1 choose other parametric value.

Step 6: Using (8.13), find $TP(T^*, g_e^*, p^*)$ value.

Step 7: Using (8.5), find Q^* value.

8.4 Real examples with numerical experiment

8.4.1 Real examples

The present study deals with retailer who sells the fresh perishable product. A farmer who sells agricultural products by growing crops like vegetables or fruits using nature-based biological fertilizers and environmentally friendly pesticides as a green investment, except using harmful chemicals and selling organic farming products to the buyers. Other example is, a beverages industry used the green packaging concept for fresh juice. Green processing technology applied in dairy products and used the green packaging for dairy products. This model will be applicable for organic agriculture products, green dairy products, green beverages products, and nutrient-dense foods type of health conscious perishable products etc.

8.4.2 Numerical experiment

To demonstrate the specified model and its solution process, numerical example is included in this section. The parameters in this section were assumed from Soni et al.[215]and found suitable for proposed model.

$\alpha = 450$ units, $\beta = 5.5$, $\gamma = 0.5$, $A = ₹500/\text{order}$, $C_d = ₹2/\text{unit}$, $h = ₹1/\text{unit}$, $C = ₹10/\text{unit}$, p_0

$= 15\%$, $\tau = \frac{90}{365}$ year, $t_d = \frac{8}{365}$ year, $\lambda = 4$.

According to the solution procedure mentioned in above section the optimal value of replenishment cycle time $T^* = 0.1886$ year, green efforts cost is $g_e^* = ₹4.44/\text{unit time}$ and selling price $p^* = ₹52.67$ per unit. The optimal total profit $TP(T^*, g_e^*, p^*)$ is **₹2307.37** per cycle and optimal ordering quantities are $Q^* = 27$ units and optimal value of green investment cost from (8.11) is **₹7.44** per unit per cycle time.

The numerical value of various parameters and optimum value of decisions variable are substitute in (8.17), (8.19), (8.22) and (8.21), we noticed the sufficient conditions are

$$\frac{\partial^2 TP}{\partial T^2} = -1.514641 \times 10^5 < 0, \frac{\partial^2 TP}{\partial g_e^2} = -4 < 0, \frac{\partial^2 TP}{\partial p^2} = -5.928685 < 0 \quad \text{and}$$

$|H| = 6.051962784 \times 10^5 > 0$ are satisfied numerically. Next we check the concavity behaviour of objective function by graphical representations.

8.4.3 Graphical authentication of the concavity of objective functions

The objective function is concave with respect to the optimum value of selling price and cycle time as demonstrated in Figure 8.1.

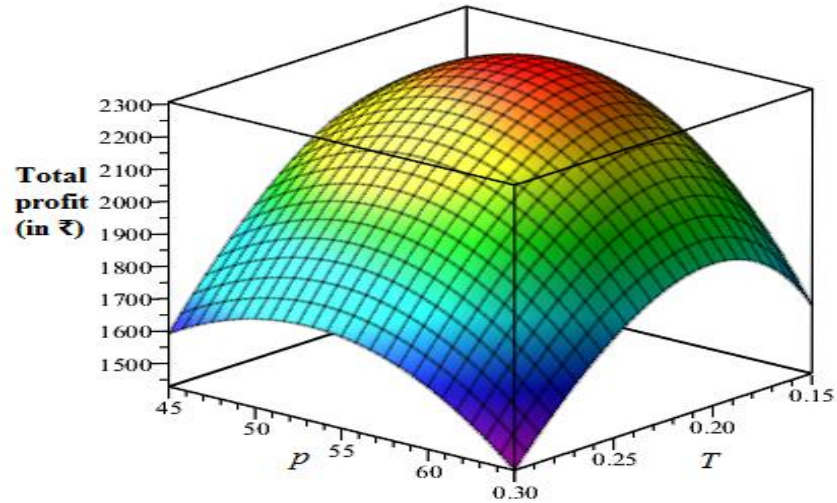


Figure 8.1 Concavity of total profit $TP(T, g_e, p)$ with respect to p and T

The objective function is concave with respect to the optimum value cycle time and greening efforts cost as mentioned in Figure 8.2.

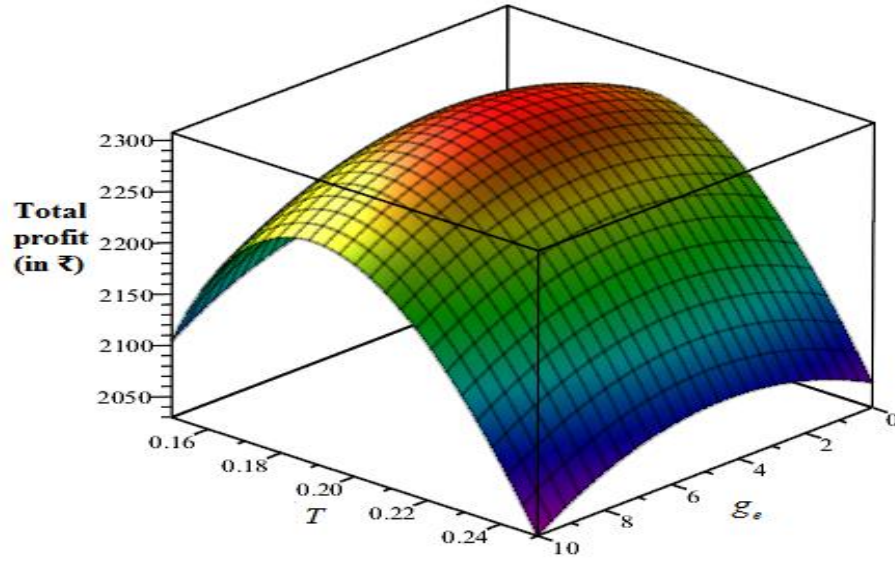


Figure 8.2 Concavity of total profit $TP(T, g_e, p)$ with respect to T and g_e

The objective function is concave with respect to the optimum value selling price and greening efforts cost as presented in Figure 8.3.

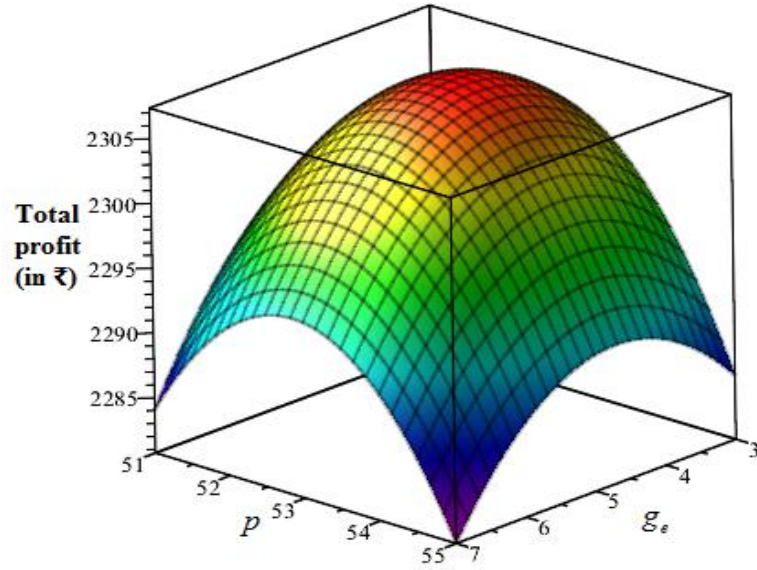


Figure 8.3 Concavity of total profit $TP(T, g_e, p)$ with respect to p and g_e

8.5 Sensitivity analysis and observations

Sensitivity analysis is carried out to investigate how different parameters impact the optimal solution of the suggested inventory model by altering each parameter from -20% to +20% individually while leaving the others unchanged.

Table 8.1 Sensitivity performance of inventory parameters

Parameter	Change %	Value	T^* Year	g_e^* (in \$)	p^* (in \$)	Q^* (units)	TP^* (in \$)
α	-20%	360	0.2439	3.41	43.45	22.37	435.54
	-10%	405	0.2126	3.92	48.08	24.74	1272.30
	10%	495	0.1696	4.95	57.24	28.05	3540.09
	20%	540	0.1542	5.46	61.79	29.26	4969.95
β	-20%	4.4	0.1645	5.73	64.32	25.49	3978.91
	-10%	4.95	0.1766	5.01	57.85	26.11	3034.90
	10%	6.05	0.2005	3.97	48.43	26.90	1734.45
	20%	6.6	0.2123	3.58	44.89	27.09	1275.45
γ	-20%	0.4	0.1886	3.54	52.55	26.50	2293.24
	-10%	0.45	0.1886	3.99	52.61	26.53	2299.89
	10%	0.55	0.1886	4.89	52.74	26.62	2315.66
	20%	0.6	0.1886	5.34	52.81	26.67	2324.75
A	-20%	400	0.1690	4.44	52.55	25.13	2866.73
	-10%	450	0.1790	4.44	52.62	25.91	2579.41
	10%	550	0.1977	4.44	52.72	27.14	2048.47
	20%	600	0.2063	4.43	52.75	27.62	1800.93
C	-20%	8	0.1835	4.57	51.51	26.96	2595.21
	-10%	9	0.1860	4.50	52.09	26.77	2449.81
	10%	11	0.1912	4.37	53.25	26.38	2167.95
	20%	12	0.1940	4.31	53.83	26.17	2031.54
C_d	-20%	1.6	0.1886	4.44	52.67	26.58	2307.71
	-10%	1.8	0.1886	4.44	52.67	26.57	2307.54
	10%	2.2	0.1886	4.44	52.67	26.57	2307.20
	20%	2.4	0.1886	4.44	52.67	26.57	2307.03
h	-20%	0.8	0.1886	4.44	52.66	26.58	2309.62
	-10%	0.9	0.1886	4.44	52.67	26.58	2308.49
	10%	1.1	0.1886	4.44	52.67	26.57	2306.22
	20%	1.2	0.1886	4.44	52.68	26.57	2305.11
p_0	-20%	0.12	0.1882	4.46	51.29	26.49	2320.91
	-10%	0.135	0.1884	4.45	51.97	26.53	2314.63
	10%	0.165	0.1888	4.43	53.38	26.62	2299.03
	20%	0.18	0.1891	4.41	54.11	26.68	2289.55
t_d	-20%	0.017534	0.1880	4.45	52.98	25.93	2200.17
	-10%	0.019726	0.1883	4.44	52.82	26.25	2254.06
	10%	0.02411	0.1889	4.43	52.52	26.89	2360.12
	20%	0.026301	0.1892	4.43	52.38	27.21	2412.25
τ	-20%	0.19726	0.1690	4.43	52.41	22.95	1818.56
	-10%	0.221918	0.1791	4.43	52.55	24.80	2081.66
	10%	0.271233	0.1976	4.44	52.77	28.27	2503.86
	20%	0.29589	0.2062	4.45	52.86	29.90	2676.98
λ	-20%	3.2	0.1886	5.56	52.75	26.63	2317.23
	-10%	3.6	0.1886	4.94	52.71	26.60	2311.75
	10%	4.4	0.1886	4.03	52.64	26.55	2303.78
	20%	4.8	0.1886	3.69	52.62	26.54	2300.81
θ	-20%	0.08	0.1886	4.44	52.66	26.55	2309.42
	-10%	0.09	0.1886	4.44	52.67	26.56	2308.41
	10%	0.11	0.1886	4.44	52.67	26.59	2306.32
	20%	0.12	0.1885	4.44	52.68	26.60	2305.30

The following observations are made on the basis of Table 8.1 mathematical and numerical analysis.

(i) *Sensitivity of demand related parameters*

- With an increase in demand parameter α , one can boost the demand and order size increases. Higher value of α produced that shorter inventory cycle, higher selling price and higher profit. This means that the retailer gets profit from the increase of the market demand by asking for higher prices and shortening the inventory cycle, which helps in reducing the holding and the deterioration cost.
- Changes in parameter β directly influence the demand, β is increases then ordering quantity, selling price and green efforts cost are decreases but cycle time increases and finally total profit decreases with β is increases.
- The higher value of greening investments effectiveness scale γ result to slightly shorter cycle time, higher prices, higher ordering quantity. Finally higher value of γ gives the higher profit.
- Shelf life of perishable product τ increases than cycle time, selling price, and green efforts cost slightly increases. The order quantity and total profit increases with increase the value of τ .
- Higher non-deteriorating period t_d have a positive impact on total profit, selling price and replenishment ordering quantity.
- Price discount facility during the deterioration period improves the market demand but if increases price discount then optimal value of selling price, cycle time, ordering quantity will be decreases with total profit decreases.

(ii) *Sensitivity of inventory cost related parameters*

- Total profit and parameters A, C, C_d, h are inversely proportional to each other. Optimal cycle time slightly increases with A and C but cycle time remain unchange with changes in C_d and h . Selling price remain unchanged with changes in A, C_d, h and purchase cost C increases then selling price also. Ordering quantity also increases with A, C_d, h but C increases the ordering quantity decreases.

(iii) *Sensitivity of parameters λ and θ*

- Greening efforts effectiveness parameter λ increases then total profit along with order quantity, selling price, and greening efforts cost decreases. Cycle time slightly increases with λ .
- With higher rates of deterioration, perishable products lose their utility value. The increase in the deterioration of products unit will have a negative impact on profit function. The higher rate of deterioration result to replenish more order quantities.

8.6 Discussion about managerial insights

From the model formulations and their optimal results, sensitivity analysis and observations following managerial insights are summarized:

- The optimum values of cycle time, green effort cost, and selling price of a product are important for a retailer to take precise decisions about when to replenish orders, how many quantities to replenish, what is a proper green investment cost, and the selling price of a product such that total profit is maximized.
- The retailer should keep up scale demand for the fresh products; higher scale demand results in higher profit with the lowest cycle time and higher product replenish quantity.
- The main factor that influences market demand is product freshness; our analysis showed that if a product has a longer shelf life in terms of a higher freshness index, the retailer will profit more from selling more of it.
- The rate of physical deterioration of the product plays an important role; a higher rate of physical deterioration slightly reduces the profit.
- Product freshness is depends on deterioration rate, at the initial stage there is no effect of deterioration. More timing of non deterioration period gives the maximum freshness level of products. So, the retailers choose to products for sells whose non deterioration period is higher.
- The physical deterioration and quality-based deterioration of products are taken into account, which is an indication for retailers to know about the degradation of products so they can decide how to properly preserve them.
- Higher value of greening efforts effectiveness parameter presented that higher green investment produces the green/organic products which is healthfulness, and

environmental friendliness. Our study shown that higher investment in greening gives the higher profit with increases quantities of products and selling price.

- The price discount facility improves market demand for products by increasing the order quantity. The selling price directly impacts demand; during the deteriorating period, products may lose their quality, resulting in decreased demand. To boost demand and clear stocks during cycle time, the price discount facility is a tool for reducing loss.
- Ordering cost, deterioration cost increase the total cost. The retailer tried to reduce the ordering cost and deterioration cost.

8.7 Conclusions

This chapter provides a combined framework for joint optimal pricing, cycle time, and greening efforts (investment) strategies for a perishable product with the objective of retailer's profit maximisation. We consider both physical deterioration at a constant rate of the existing inventory, and a degradation of the product's freshness quality over time. One of the novel ideas of this is that every perishable product may not deteriorate physically as well as qualitatively more rapidly at the start of their shelf life, and their freshness does not degrade as noticeably. As a result, demand during a non deteriorating period is price and green efforts depended taken. After a certain period, products affected by deterioration of both type, product value degrade continuously; during this period, demand is determined by freshness, price, and greening efforts; and the retailer offers a price discount during the deterioration-affected period to stimulate demand. None of the researchers adopted the concept of greening efforts with freshness and price-related demand which is unique idea of this study. Organic agriculture products, green packaged beverage products, and green dairy products, whose demand not determined only by freshness and price, but also by the consumer's preference for greenness. For clarification of the model, the problem was formulated into a mathematical model, and a solution procedure was given with an example. We use sensitivity analysis to demonstrate the analytical findings and provide relevant management implications as a conclusion. The result shows that a higher investment in greening and a longer shelf life of products with a minimum deterioration period increase the total profit. The findings of this study can be used to inform decisions about the control of perishable inventory that take freshness, greening efforts, and deterioration of the products into account.

The model presented in this article has some limitations like product physical deterioration is taken constant but most of product nature has a time dependent deterioration rate, another limitation is that inventory level reaches zero at end of cycle, it is not always true for perishable products inventory system. The possible extension of this model is to be considering some stocks remaining at end of inventory cycle time and optimize the stock level of remaining inventory. The preservation technology concept use for reducing the deterioration, Model may be expanding with different payment policy and concept of carbon emission with carbon tax, cap and trade, carbon limit policy. In additional, the proposed model can be generalized by allowing shortages.

CHAPTER-9

An EPQ model for Delay Deteriorating Perishable Products with Price, Freshness and Greening Efforts Dependent Demand under Markdown Strategy

9.1 Introduction

A decision-maker or producer employs various business techniques to boost their profit. Greening efforts are an action taken by a decision-maker or producer to minimize the impact trade has on the ecosystem and ensure sustainable products. The market demand is significantly influenced by the product's freshness, greening level, selling price and deterioration. The quality of a product is greatly determined by how recently it was produced; hence the freshness of green products has an impact on consumer purchase decisions. Consumers today use green, fresh, perishable products because of their freshness, healthfulness, and sustainability. In this chapter, we developed the continuous production inventory model for the producer who produces and sells fresh perishable products with the input of green efforts. There are two distinct kinds of product decay to take into account: products whose physical condition gradually deteriorates over time at a constant rate, and products whose freshness quality declines with time. It has been observed that the deterioration effect is negligible at the start of the production period for perishable products, and the freshness of the product does not decrease as noticeably during this time. i.e., the product is fully fresh. After production stops, the effect of deterioration starts. Due to the effect of deterioration, the product loses its freshness continuously, so market demand decreases, and hence policymakers adopt a markdown strategy after some time of deterioration to stimulate demand. In order to increase the sales

of inventory and enhance the profit from clearing stocks at the end of their life, we have adopted the markdown policy. Taking into account all of these factors, demand for perishable products is a function of selling price, age of the product (freshness), and greening efforts. The main challenges of the proposed chapter are: (i) what is the optimum value of the cycle time, greening level, and markdown percentages such that the producer's total profit is the maximum?. (ii) When to start and stop production? When does the decision-maker apply the markdown offer?. (iii) How does product freshness affect total profit? (iv) What is the role of green investment in terms of order quantity and profit?. (v) What is the contribution of the markdown policy?. The explanations of the challenges and other significant results of this study are given in the managerial insights and conclusion section. The study's goals are to determine the optimum duration for replenishment cycle time, the optimum cost of greening efforts, and the optimum markdown percentage in order to maximize the producer's total profit. A mathematical formulation that reflects real-world circumstances is validated with a numerical example. To assess the model's stability, the parameters are analyzed according to sensitivity analysis. A discussion of the future direction of research is included in the article's conclusion, which also includes some noteworthy managerial insights as significant result of this chapter.

9.2 Notations and Assumptions

The framework of the proposed chapter contains the following notations and assumptions.

9.2.1 Notations

Parameters

A	Set up cost per cycle (in ₹/Set up)
h	Constant holding cost (₹/unit/unit time).
p	Original price of product (₹/unit)
r	Markdown rate; defined as the percentage decrease of an original price of product.
P	Production rate proportional to demand (unit/year).
k	Production cost per unit per cycle. (₹/unit)
δ_p	Production percentage.

τ	Maximum life-time of the product beyond which no consumer will buy the product. $0 < T \leq \tau$
λ	Greening efforts effectiveness parameter. ($\lambda > 0$)
θ	Rate of deterioration of product. $0 < \theta < 1$.
C_d	Deterioration cost (unit/year)
l_p	proportional factor of production rate and demand, $l_p \geq 1$

Decision variables

T	Cycle time; (in years); where $T_1 + T_2 + T_3 = T \leq \tau$
g_e	Cost of greening efforts; (in ₹/unit).
m_p	Markdown percentage.

Objective function

$TP(T, g_e, m_p)$ Producer's total profit per cycle (in ₹).

Expressions and functions

$R(p, t, g_e)$	Demand function at time t .
$I_1(t)$	Inventory level at time t during $0 \leq t \leq T_1$.
$I_2(t)$	Inventory level at time t during $0 \leq t \leq T_2$.
$I_3(t)$	Inventory level at time t during $0 \leq t \leq T_3$.
Q_1	Inventory level (total production) at time T_1 (units).
Q_2	Total quantity under markdown offered after time T_2 (units), $Q_2 \leq Q_1$.
T_1	Production period; (in years)
$T_1 + T_2$	Markdown offering time; (in years)
T_3	Markdown period; (in years)

9.2.2 Assumptions

1. A single type of perishable product is considered over a specific cycle time.
2. The rate of production is proportional to demand i.e. $P = l_p \cdot R(p, t, g_e)$, $0 \leq t \leq T$ and production stops at time T_1 . After a product unit has been produced, it must be sold. (Shah and Vaghela [58])
3. Production time is proportional to the cycle time which is equivalent to $T_1 = \delta_p T$.

4. A product starts to deteriorate after production stops, and a markdown is offered after some time of product deterioration. Products have two kinds of deterioration, physical deterioration at a constant rate of available inventory and deterioration in the freshness quality of the product.
5. Deteriorated products cannot be repaired or replaced, and they have no salvage value.
6. Markdown pricing is applied one time during a cycle.
7. The markdown price is known, in advance.
8. A number of elements such as time spent on resources, rate of warmth, temperature, and preservation, among others, may have an influence on the product's freshness. It appears to be impossible to obtain a product's explicit freshness level. Nevertheless, it goes without mentioning that any product's freshness gradually degrades and wears out over time. Therefore, we can assume that the freshness index is 1 at time 0 and gradually decreases over time until it eventually reaches 0 as the product gets closer to its expire (i.e, it cannot be sold).

The freshness index is defines as $f(t) = \frac{\tau - t}{\tau}, 0 \leq t \leq \tau$. (Chen et al.[75],

Dobson[211], Agi and Soni[212], Soni[215]).

9. The replenishment cycle time is shorter than the longest possible product shelf life. i.e. $T \leq \tau$.
10. The producer's greening effort requires a capital investment in greening efforts over a certain time frame rather than raising the unit price of the product. The producer uses green investments to produce or maintain organic or green products.

11. Total greening investments per unit time t is $\int_0^t \int_0^{g_e} \lambda \cdot g_e dg_e dt = \frac{\lambda \cdot g_e^2 \cdot t}{2}$. So, the total

investment for the time duration T is $\frac{\lambda \cdot g_e^2 \cdot T}{2}$. (Swami and Shah[227], Raza and

Faisal[229], Shah et al.[28],Shah et al.[232]).

12. The demand rate is based on selling price of product, freshness index, i.e. age of product as well as green efforts. Demand rate function mathematically defined as,

$$R(p, t, g_e) = \begin{cases} r(p) + \gamma g_e, & 0 \leq t \leq T_1 \\ r(p)f(t) + \gamma g_e, & 0 \leq t \leq T_2 \\ r(p_r)f(t) + \gamma g_e, & 0 \leq t \leq T_3 \end{cases} \text{ in this expression } r(p) = \alpha - \beta p \text{ and}$$

$r(p_r) = \alpha - \beta p(1-r)$,are any non-negative, continuous, convex decreasing functions of selling price, and $\alpha > 0$ represent the constant market demand , $\beta > 0$ is the price elasticity factor, $\gamma > 0$ is greening investments effectiveness scale and $T_1 + T_2 + T_3 = T \leq \tau$.

13. The lead time is negligible and the replenishment rate infinite and shortages are not permissible.

9.3 Mathematical Formulation

Based on the assumptions and notations, the behaviour of the inventory system depicted in Figure 9.1.

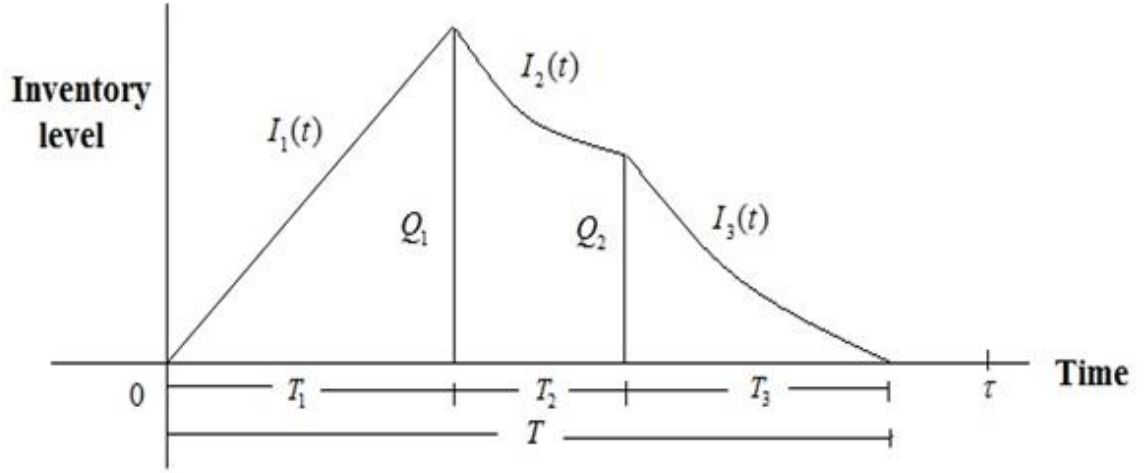


Figure 9.1 Behaviour of the inventory system

From the Figure 9.1, at the time $t = 0$ the inventory level is zero. Production and supply of the fresh green product begin concurrently, and production ends at $t = T_1$ at which maximum inventory level Q_1 is reached. During $0 \leq t \leq T_1$, there is no physical deterioration and freshness index of product is $f(t) \rightarrow 1$, i.e. product is fully fresh. The inventory level during $0 \leq t \leq T_1$ can be represented by the following differential equation,

$$\frac{dI_1(t)}{dt} = P - (r(p) + \gamma g_e) = (l_p - 1)(r(p) + \gamma g_e), 0 \leq t \leq T_1 \quad (9.1)$$

With the condition $I_1(0) = 0$, the solution of (9.1) is,

$$I_1(t) = (l_p - 1)(r(p) + \gamma g_e)t \quad (9.2)$$

The inventory level at time $t = T_1$ is i.e. $I_1(T_1) = Q_1$;

$$Q_1 = (l_p - 1)(r(p) + \gamma g_e)T_1 \quad (9.3)$$

After time $t = T_1$, inventory level decrease due to demand and physical deterioration as well as freshness degradation of product. The inventory level on $0 \leq t \leq T_2$ is,

$$\frac{dI_2(t)}{dt} + \theta I_2(t) = -(r(p)f(t) + \gamma g_e), 0 \leq t \leq T_2 \quad (9.4)$$

The solution of (9.4) at $I_2(0) = Q_1$ is given by;

$$I_2(t) = Q_1 e^{-\theta t} + (e^{-\theta t} - 1) \left(\frac{\gamma g_e}{\theta} + \frac{r(p)}{\theta} + \frac{r(p)}{\tau \theta^2} \right) + \frac{r(p)t}{\tau \theta} \quad (9.5)$$

Likewise, at $0 \leq t \leq T_3$, level of inventory declines due to joint effect of demand and deterioration. The demand is also decrease with time. Markdown policy is applied during this interval to boost demand through a reduction in the original selling price. Thus, we have

$$\frac{dI_3(t)}{dt} + \theta I_3(t) = -(r(p_r)f(t) + \gamma g_e), 0 \leq t \leq T_3 \quad (9.6)$$

Finally, the solution of (9.6) using boundary condition $I_3(0) = Q_2$,

$$I_3(t) = Q_2 e^{-\theta t} + (e^{-\theta t} - 1) \left(\frac{\gamma g_e}{\theta} + \frac{r(p_r)}{\theta} + \frac{r(p_r)}{\tau \theta^2} \right) + \frac{r(p_r)t}{\tau \theta} \quad (9.7)$$

Noted that $I_3(T_3) = 0$; the inventory level at $t = T_2$ is,

$$Q_2 = (e^{\theta T_3} - 1) \left(\frac{\gamma g_e}{\theta} + \frac{r(p_r)}{\theta} + \frac{r(p_r)}{\tau \theta^2} \right) - \frac{r(p_r)T_3}{\tau \theta} e^{\theta T_3} \quad (9.8)$$

Now, calculate the different inventory costs and sales revenue, to find the total profit of producer.

$$\text{Annual fixed setup cost: } STC = \frac{A}{T} \quad (9.9)$$

$$\text{Production cost per unit time: } PDC = \frac{k}{T} \int_0^{T_1} l_p \cdot (r(p) + \gamma g_e) dt \quad (9.10)$$

The holding cost of holding inventory per unit time:

$$HC = \frac{h}{T} \left[\int_0^{T_1} I_1(t) dt + \int_0^{T_2} I_2(t) dt + \int_0^{T_3} I_3(t) dt \right] \quad (9.11)$$

Deterioration cost per unit time:

$$DC = \frac{C_d}{T} \left[\left(Q_1 - \int_0^{T_2} I_2(t) dt \right) + \left(Q_2 - \int_0^{T_3} I_3(t) dt \right) \right] \quad (9.12)$$

Depending on the product's level of greening, producers may have to incur extra expenses to increase the product's quality. Greening efforts investment per cycle is,

$$GEI = \frac{1}{T} \int_0^T \int_0^{g_e} \lambda \cdot g_e dg_e dt = \frac{1}{T} \frac{\lambda \cdot g_e^2 \cdot T}{2} = \frac{\lambda \cdot g_e^2}{2} \quad (9.13)$$

The revenue before markdown and the revenue after markdown are combined to form the total revenue. Hence, the total sales revenue generated by each cycle is given as,

$$SR = \frac{p}{T} \left(\int_0^{T_1} (r(p) + \gamma g_e) dt + \int_0^{T_2} (r(p)f(t) + \gamma g_e) dt \right) + \frac{p(1-r)}{T} \int_0^{T_3} (r(p_r)f(t) + \gamma g_e) dt \quad (9.14)$$

The total profit of the producer per cycle time T is formulated as,

$$TP_1(T_1, T_2, T_3, g_e, m_p) = SR - (STC + PDC + HC + DC + GEI) \quad (9.15)$$

$$TP_1(T_1, T_2, T_3, g_e, m_p) =$$

$$\begin{aligned} & \frac{p}{T} \left(\int_0^{T_1} (r(p) + \gamma g_e) dt + \int_0^{T_2} (r(p)f(t) + \gamma g_e) dt \right) \\ & + \frac{p(1-r)}{T} \int_0^{T_3} (r(p_r)f(t) + \gamma g_e) dt \\ & - \left(\frac{A}{T} + \frac{k}{T} \int_0^{T_1} l_p \cdot (r(p) + \gamma g_e) dt + \frac{h}{T} \left(\int_0^{T_1} I_1(t) dt + \int_0^{T_2} I_2(t) dt + \int_0^{T_3} I_3(t) dt \right) \right. \\ & \left. + \frac{C_d}{T} \left(\left(Q_1 - \int_0^{T_2} I_2(t) dt \right) + \left(Q_2 - \int_0^{T_3} I_3(t) dt \right) \right) + \frac{1}{T} \int_0^T \int_0^{g_e} \lambda \cdot g_e dg_e dt \right) \end{aligned}$$

Equation (9.15) is the form of T_1 , T_2 and T_3 but as per Srivastava and Gupta[238] the relations of T_1 , T_2, T_3 with δ_p, m_p and T defines as,

$$T_1 = \delta_p T,$$

$$T_2 = m_p(T - T_1) = m_p(1 - \delta_p)T \text{ and}$$

$$T_3 = T - (T_1 + T_2) = (1 - \delta_p)(1 - m_p)T$$

Substitute above relations in (9.15), it can be rewritten as in form of T is,

$$TP(T, g_e, m_p) = SR - (STC + PDC + HC + DC + GEI) \quad (9.16)$$

Due to complexity of nonlinear form of (9.16), to find the value of decision variables T^* , g_e^* , m_p^* , and to prove the concavity of total profit function $TP(T^*, g_e^*, m_p^*)$, we adopted following solution procedure.

9.3.1 Solution technique to determine the optimal solution

In this section, we determine the optimal value of decision variables T^* , g_e^* , m_p^* which maximize the total profit $TP(T^*, g_e^*, m_p^*)$ per cycle. The necessary conditions for maximize of the total profit function given by (9.16) are,

$$\frac{\partial TP}{\partial T} = 0, \frac{\partial TP}{\partial g_e} = 0, \frac{\partial TP}{\partial m_p} = 0 \quad (9.17)$$

Use the Hessian matrix method, to prove the concavity of total profit function $TP(T^*, g_e^*, m_p^*)$ at the value of decision variables T^* , g_e^* , m_p^* . Let's take third order Hessian matrix,

$$H(T^*, g_e^*, m_p^*) = \begin{bmatrix} \frac{\partial^2 TP(T^*, g_e^*, m_p^*)}{\partial T^2} & \frac{\partial^2 TP(T^*, g_e^*, m_p^*)}{\partial T \partial g_e} & \frac{\partial^2 TP(T^*, g_e^*, m_p^*)}{\partial T \partial m_p} \\ \frac{\partial^2 TP(T^*, g_e^*, m_p^*)}{\partial g_e \partial T} & \frac{\partial^2 TP(T^*, g_e^*, m_p^*)}{\partial g_e^2} & \frac{\partial^2 TP(T^*, g_e^*, m_p^*)}{\partial g_e \partial m_p} \\ \frac{\partial^2 TP(T^*, g_e^*, m_p^*)}{\partial m_p \partial T} & \frac{\partial^2 TP(T^*, g_e^*, m_p^*)}{\partial m_p \partial g_e} & \frac{\partial^2 \pi(T^*, g_e^*, m_p^*)}{\partial m_p^2} \end{bmatrix} \quad (9.18)$$

Conditions for concavity at (T^*, g_e^*, m_p^*) are,

$$\frac{\partial^2 TP}{\partial T^2} < 0, \frac{\partial^2 TP}{\partial T^2} \cdot \frac{\partial^2 TP}{\partial g_e^2} - \left(\frac{\partial^2 TP}{\partial T \partial g_e} \right)^2 > 0 \text{ and } \det(H) < 0 \quad (9.19)$$

Furthermore, to verify the all Eigen values of (9.18) are negative then $TP(T^*, g_e^*, m_p^*)$ maximize. (Cardenas-Barron and Sana [203]).

To obtain an optimal solution of decision variables and optimal profit function, follow the steps mentioned below:

Step 1: First allocate value of inventory parameters with proper unit other than decision variables.

Step 2: Take the partial derivative of (9.16) with respect to T , g_e and m_p and equating to zero.

Step 3: Solving the equations stated in (9.17) simultaneously using the mathematical software Maple XVIII, to find T^* and g_e^* and m_p^*

Step 4: Verify the sufficient conditions stated in (9.19) at T^* and g_e^* and m_p^* , if not satisfied go to step 1 and choose other value of parameters in step 1, repeat process till (9.19) satisfied.

Step 5: Using (9.16), find $TP(T^*, g_e^*, m_p^*)$ value.

Step 6: Using (9.3) and (9.8), find Q_1^* and Q_2^* value.

Step 7: stop

9.4 Real examples with numerical experiment

9.4.1 Real examples

The proposed model concerns the producer, who produces and sells the fresh, green, perishable product. Let's take the real examples: a beverage industry produced a fresh, nutrient-dense juice with green packaging. At the time of production, sealed juice is absolutely fresh and exhibits no indications of deterioration; however, after some time, packaged juice can display signs of deterioration, and the company that produces it may apply a markdown policy to raise demand. Juice producers apply a "best before date" policy. Green processing technology was applied to dairy products, and green packaging was used for perishable dairy products. This model is suitable for the beverage industry, dairy industry, and pharmaceutical sectors for perishable products. Another practical example is how farmers produce and sell organic fruits and vegetables without using hazardous pesticides or artificial fertilizers. Instead, they make a green investment in bio-

pesticides and organic fertilizers based on the natural world to meet consumer demand for organic, fresh foods. This model may be relevant to perishable goods with low market demand relative to other goods and ongoing quality decline over time.

9.4.2 Numerical experiment

To demonstrate the findings of the proposed study, the following parametric values are used in a numerical example.

$\alpha = 450$ units, $\beta = 3.3$, $\gamma = 0.5$, $l_p = 1.5$, $A = ₹200/\text{order}$, $C_d = ₹10/\text{unit}$, $h = ₹3/\text{unit}$, $k = ₹2/\text{unit}$, $\delta_p = 0.4$, $r = 5\%$, $\tau = \frac{180}{365}$ year, $p = ₹100/\text{unit}$, $\lambda = 5$, $\theta = 0.08$. Optimal results derived as per the steps mentioned in previous section. The optimal value of decision variables is $T^* = \mathbf{0.28411}$ year, green efforts cost is $g_e^* = \mathbf{₹9.60/\text{unit}}$ time, optimal markdown percentage is $m_p^* = 0.53197$. The optimal **total profit** $TP(T^*, g_e^*, m_p^*)$ is **₹10805.76** per cycle time, optimal **production period** is $T_1^* = 0.1136$ year. Optimal **markdown offering** from $T_1^* + T_2^* = 0.2043$ year and **markdown period** $T_3^* = 0.0798$ year. Now from (9.18) and (9.19), hessian matrix at optimal value of decision variables is,

$$H(T^*, g_e^*, m_p^*) = \begin{bmatrix} -17433.9 & -0.309320 & -662.939832 \\ -0.309320 & -5 & 4.571022540 \\ -662.939832 & 4.571022540 & -5433.546669 \end{bmatrix}, \quad \frac{\partial^2 TP}{\partial T^2} = -17433.9 < 0$$

$$\frac{\partial^2 TP}{\partial T^2} \cdot \frac{\partial^2 TP}{\partial g_e^2} - \left(\frac{\partial^2 TP}{\partial T \partial g_e} \right)^2 = 87169.40 > 0, \quad \det(H) = -4.7979 < 0 \text{ and Eigen values of}$$

Hessian matrix are $\lambda_1 = -17470.41 < 0$, $\lambda_2 = -4.99 < 0$, $\lambda_3 = -5397.03 < 0$. Hence, it is proved that the optimal value of decision variables satisfied sufficient conditions of concavity. Here noticed that, the original selling price ($p = ₹100/\text{unit}$) and markdown rate ($r = 5\%$) are known, it means that markdown price ($p(1-r) = \mathbf{₹95}$) is also known.

9.4.3 Graphical authentication of the concavity of objective functions

The graphical representation of concavity of the objective function as mentioned in Figure 9.2, Figure 9.3 and Figure 9.4 as below;

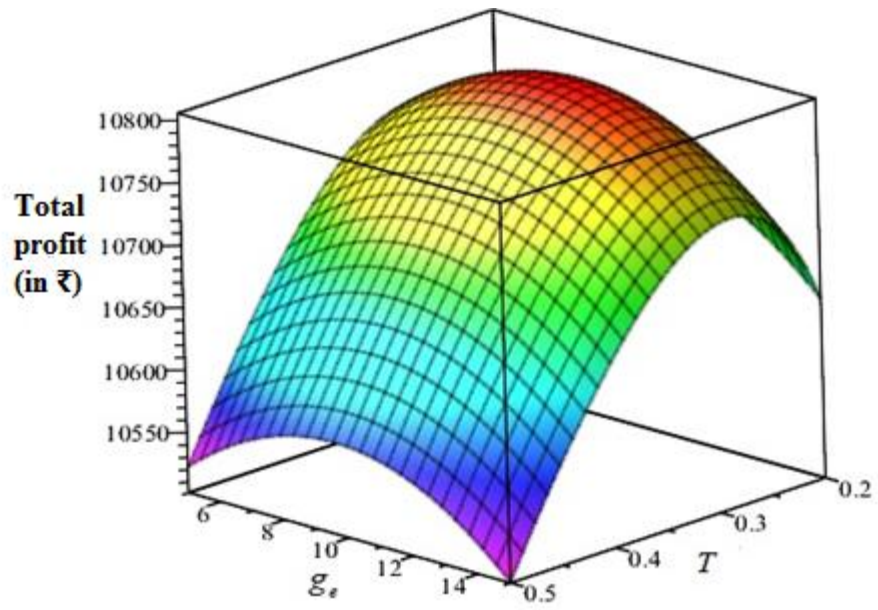


Figure 9.2 Concavity of total profit $TP(T, g_e, m_p)$ with respect to g_e and T

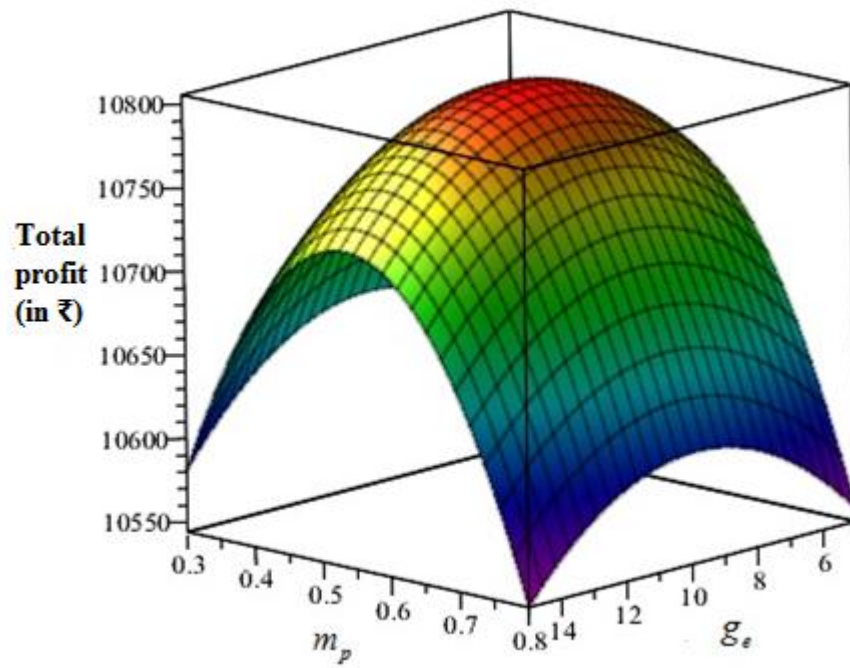


Figure 9.3 Concavity of total profit $TP(T, g_e, m_p)$ with respect to m_p and g_e

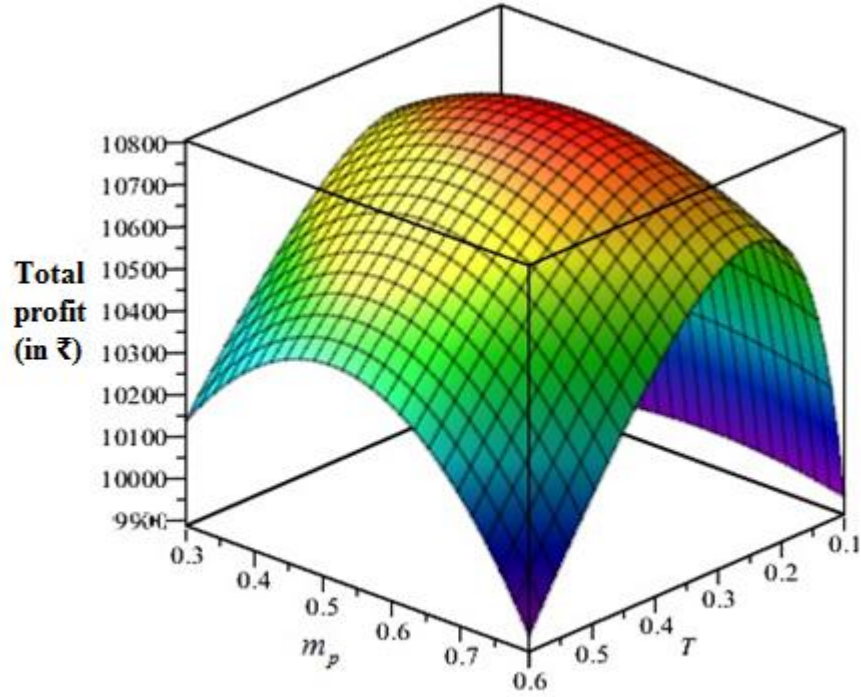


Figure 9.4 Concavity of total profit $TP(T, g_e, m_p)$ with respect to m_p and T

9.5 Sensitivity analysis and discussion:

The suggested inventory model's optimum solutions are examined using sensitivity analysis, which changes each parameter from -40% to +40% individually while leaving the rest untouched. The following tabular values give the changes in decisions variable corresponding to change in inventory parameters.

- **Effect of markdown rate r**

Table 9.1 Variations effect of markdown rate on decisions variables and total profit

r	T^*	g_e^*	m_p^*	Q_1^*	Q_2^*	TP^*	$T_1^* + T_2^*$	T_3^*
0.03	0.2837	9.68	0.5653	14.17	9.28	10716.92	0.2097	0.0740
0.04	0.2841	9.64	0.5482	14.17	9.86	10761.41	0.2071	0.0770
0.05	0.2841	9.60	0.5320	14.18	10.42	10805.76	0.2043	0.0798
0.06	0.2837	9.56	0.5166	14.19	10.97	10849.78	0.2014	0.0823
0.07	0.2829	9.51	0.5020	14.20	11.50	10893.27	0.1984	0.0845

From Table 9.1, we observed that, if we increase the markdown rate by -40% to +40%, then total profit increases 1% to 2%, markdown percentage and production quantity reduces decreases slightly. Markdown offering time $T_1 + T_2$ will be reduced and markdown period T_3 slightly increases with number of markdown quantities increases. Markdown percentage decreases with increases of markdown rate.

- **Effect of production percentage δ_p**

Table 9.2 Variations effect of production percentage on decisions variables and total profit

δ_p	T^*	g_e^*	m_p^*	Q_1^*	Q_2^*	TP^*	$T_1^* + T_2^*$	T_3^*
0.24	0.22644	9.88	0.534336	-	-	10904.34	0.1463	0.080138
0.32	0.25212	9.74	0.533264	10.70	10.46	10855.65	0.1721	0.080018
0.4	0.28411	9.60	0.53197	14.18	10.42	10805.76	0.2043	0.079783
0.48	0.324909	9.46	0.530366	19.45	10.37	10754.15	0.2456	0.089607
0.56	0.378398	9.31	0.528311	26.42	10.26	10699.88	0.2999	0.087961

Table 9.2 indicated that, if we increase the production percentage by -40% to +40%, then it is obvious that production quantity increases with total profit decreases and total cycle time increases. Mark down percentage and green efforts will be reduces with higher value of δ_p .

- **Effect of maximum life time τ**

Table 9.3 Variations effect of maximum life time on decisions variables and total profit

τ	T^*	g_e^*	m_p^*	Q_1^*	Q_2^*	TP^*	$T_1^* + T_2^*$	T_3^*
0.2959	0.2217	9.5968	0.5229	11.06	8.06	10407.91	0.0887	0.0696
0.3945	0.2551	9.5990	0.5276	12.73	9.34	10644.65	0.1020	0.0808
0.4932	0.2841	9.6012	0.5320	14.18	10.42	10805.76	0.1136	0.0907
0.5918	0.3101	9.6032	0.5360	15.48	11.38	10924.41	0.1240	0.0997
0.6904	0.3337	9.6053	0.5398	16.66	12.22	11016.38	0.1335	0.1081

Table 9.3 depicted that if we increase the life of product (freshness) then total profit will be increases. The production quantity and markdown offering quantity increases with increases the life of product. Markdown offering time will be late because product freshness is increasing. Green efforts cost increases with increases the value of τ .

- **Effect of constant demand α , price elasticity factor β , and green investment effectiveness scale γ**

Table 9.4 and Table 9.5, shown that α and γ increases by -40% to -40% then total profit increases. The production quantity and markdown offering quantity also increases with increases α and γ . Markdown offering time and markdown duration reduces with total cycle time reduces as α increases but Markdown offering time increases and markdown duration decreases with total cycle time reduces as γ increases. Greening efforts cost and markdown percentage increases due to increases α and γ .

Table 9.4 Variations effect of constant demand on decisions variables and total profit

α	T^*	g_e^*	m_p^*	Q_1^*	Q_2^*	TP^*	$T_1^* + T_2^*$	T_3^*
270	0.48253	9.415253	0.344365	6.70	8.08	2844.26	0.2927	0.1898
360	0.352416	9.535272	0.464781	11.24	9.75	6764.70	0.2392	0.1132
450	0.28411	9.601177	0.53197	14.18	10.42	10805.76	0.20433	0.0798
540	0.209662	9.695342	0.630369	18.02	10.90	19060.96	0.1631	0.0465
630	0.166498	9.774247	0.71459	20.31	11.01	27455.17	0.1380	0.0285

Table 9.5 Variations effects of green investment effectiveness scale on decisions variables and total profit

γ	T^*	g_e^*	m_p^*	Q_1^*	Q_2^*	TP^*	$T_1^* + T_2^*$	T_3^*
0.3	0.2844	5.7578	0.5268	13.85	10.29	10658.34	0.2036	0.0807
0.4	0.2843	7.67880	0.52910	13.99	10.35	10722.82	0.2039	0.0803
0.5	0.2841	9.6012	0.5320	14.18	10.42	10805.76	0.2043	0.0798
0.6	0.2840	11.5253	0.5356	14.41	10.51	10907.20	0.2048	0.0791
0.7	0.2836	13.4517	0.5398	14.68	10.61	11027.17	0.2053	0.0783

Total profit is negative proportional to the price elasticity factor β as per the Table 9.6. The higher value of β indicated the late markdown offer time, higher markdown period. Replenishment cycle time and markdown quantity will be increases with β but greening efforts cost and markdown percentage decreases.

Table 9.6 Variations effect of price elasticity factor on decisions variables and total profit

β	T^*	g_e^*	m_p^*	Q_1^*	Q_2^*	TP^*	$T_1^* + T_2^*$	T_3^*
1.98	0.1799	9.7862	0.7285	18.48	7.60	22854.91	0.1506	0.0293
2.64	0.2225	9.6979	0.6340	16.99	9.51	16775.48	0.1737	0.0489
3.3	0.2841	9.6012	0.5320	14.18	10.42	10805.76	0.2043	0.0798
3.96	0.3870	9.4553	0.3804	-	-	5009.61	0.2431	0.1439
4.62	0.4142	9.2151	0.1226	-	-	2260.80	0.2961	0.2181

- **Effect of cost parameters A, k, h, C_d**

Table 9.7 Variations effect of production set up cost on decisions variables and total profit

A	T^*	g_e^*	m_p^*	Q_1^*	Q_2^*	TP^*	$T_1^* + T_2^*$	T_3^*
120	0.219711	9.614363	0.542071	10.97	8.05	11123.34	0.1593	0.060367
160	0.253956	9.606791	0.536077	12.68	9.32	10954.44	0.1833	0.070689
200	0.28411	9.601177	0.53197	14.18	10.42	10805.76	0.2043	0.079783
240	0.311363	9.596702	0.528923	15.54	11.40	10671.42	0.2234	0.088005
280	0.336421	9.592965	0.526543	16.79	12.28	10547.92	0.2409	0.095569

Table 9.8 Variations effect of deterioration cost on decisions variables and total profit

C_d	T^*	g_e^*	m_p^*	Q_1^*	Q_2^*	TP^*	$T_1^* + T_2^*$	T_3^*
6	0.287212	9.602465	0.48559	14.34	11.48	10866.52	0.1986	0.088647
8	0.286062	9.599046	0.508842	14.28	10.96	10834.67	0.2018	0.084301
10	0.28411	9.601177	0.53197	14.18	10.42	10805.76	0.2043	0.079783
12	0.281384	9.609057	0.555216	14.05	9.86	10779.79	0.2063	0.075093
14	0.277903	9.622996	0.578817	13.87	9.27	10756.79	0.2077	0.070229

As per the Table 9.7 and Table 9.8, observed that the total profit slightly decreases with cost parameters A and C_d . Production quantity increases with A but decreases if increases C_d . Markdown offer time proportional to the A and C_d both but markdown period increases with A but decreases with C_d . A higher value A causes higher values of T^* but a higher value C_d causes lower values of T^* . Total profit, production quantity and mark down quantity decreases with the increases the value of k and h .

- **Effect of rate of deterioration θ**

Table 9.9 Variations effect of deterioration rate on decisions variables and total profit

θ	T^*	g_e^*	m_p^*	Q_1^*	Q_2^*	TP^*	$T_1^* + T_2^*$	T_3^*
0.048	0.2842	9.6012	0.5316	14.19	10.42	10806.17	0.2043	0.0799
0.064	0.2842	9.6012	0.5318	14.19	10.42	10805.97	0.2043	0.0799
0.08	0.2842	9.6012	0.5320	14.18	10.42	10805.76	0.2043	0.0798
0.096	0.2841	9.6012	0.5322	14.18	10.43	10805.54	0.2043	0.0797
0.112	0.2840	9.6012	0.5323	14.18	10.43	10805.32	0.2043	0.0797

From Table 9.9, increases the percentage of deterioration rate then total profit decreases. Other decision variables value almost unchanged with fluctisonous in θ .

- **Effect of Markdown price**

Table 9.10, gives the decision policy with respect to markdown price. Higher markdown price resulted to lower profit but lower markdown price gives to higher Q_2^* .

Table 9.10 Effect of Markdown price in decision strategy

$p(1-r)$	T^*	g_e^*	m_p^*	Q_1^*	Q_2^*	TP^*	$T_1^* + T_2^*$	T_3^*
75	0.2536	8.62	0.3303	12.61	19.01	11453.31	0.1517	0.1018
85	0.2702	9.13	0.4052	13.46	15.24	11206.52	0.1738	0.0964
95	0.2841	9.60	0.5320	14.18	10.42	10805.76	0.2043	0.0798
97	0.2837	9.68	0.5653	14.17	9.27	10716.92	0.2097	0.0739

Figure 9.5, shows that that the total profit highly increases with increases the constant market demand and highly decreases if the selling price elasticity parameter increases. Markdown rate increases then total profit slightly increases. Product freshness increases then total profit also increases. Cost related parameters inversely proportional to the profit.

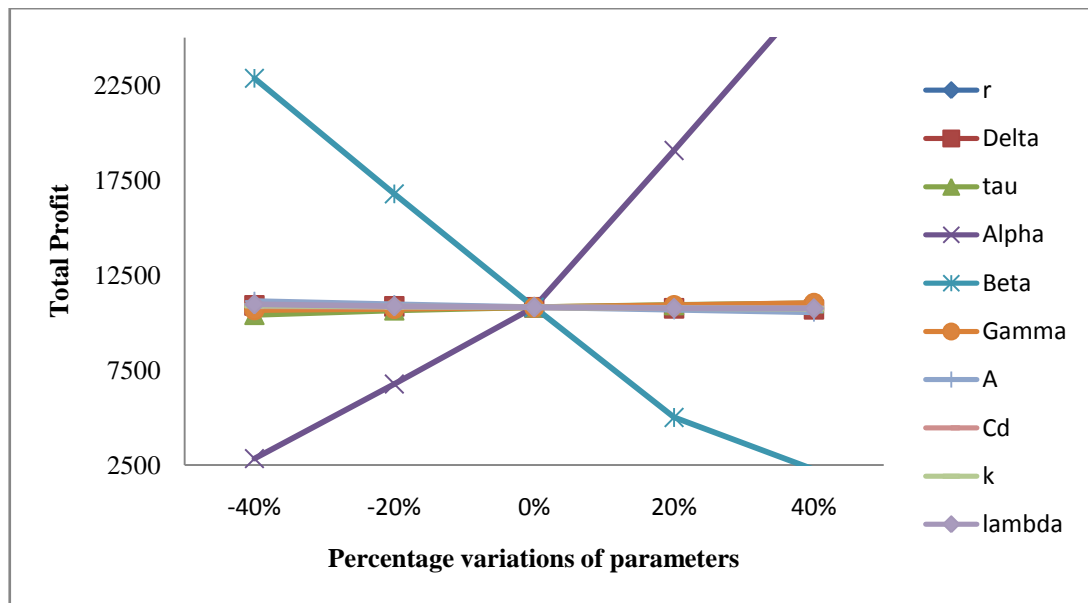


Figure 9.5 Effect of inventory key parameters on producer's total profit

9.6 Discussion about managerial insights

From the above sensitivity and mathematical analysis, following managerial insights summarized:

- The optimum value of cycle time gives the optimal value of production time, markdown offering time, markdown period, and product deterioration duration. A decision-maker can decide when to start and stop production, when to apply the markdown policy, and how much time to markdown.
- The optimal value of ordering a quantity of fresh product; and product with markdown, suggests to the decision-maker how much quantity should be replenishment per cycle such that total profit is maximized.
- A higher markdown rate results in a higher profit. If the markdown price applies to more quantity, then total profit also increases, subject to the markdown offering time and markdown period. (Table 9.1). A decision-maker should take a higher markdown rate for gain more profit.
- The producer should maintain constant demand for the fresh product, higher constant demand results in higher profit with the lowest cycle time and higher product quantity.
- Product freshness is the key determinant of market demand; our investigation revealed that if a product has a longer shelf life, the producer will make more revenue by selling more of it. (Table 9.3)

- Product freshness is based on the pace of degradation; at the beginning, deterioration has no impact. The products have a higher level of freshness when the non-deterioration period is longer (i.e., when the markdown offer is made later). So, the producer chooses to sell products whose non deterioration period is longer.
- Markdown percentage optimization helps the decision maker decide when to apply the markdown policy and how long the markdown period is so that total profit is maximized.
- Higher green investment results in green or organic products, which are more nutritious and sustainable. This is measured by the success of greening activities. Our research demonstrates that more investments in greening result in larger profits, which are increased by greater product volumes and higher markdown percentages. (Table 9.5)
- Higher production setup costs, deterioration costs, holding costs, and production costs increase the total cost of the system and reduce profit. A decision maker should try to control the different costs, ensure the system works smoothly, and increase the total profit.(Table 9.7,Table 9.8)
- The rate of physical deterioration of the product plays an important role; a higher rate of physical deterioration slightly reduces the profit. (Table 9.9)

9.7 Conclusion

In this chapter, an economic production quantity model with price, freshness level, and greening efforts dependent demand has been designed under the markdown policy. The optimal replenishment time, optimal production quantities, optimal markdown offer quantities, optimal production period, optimal markdown offering time, and optimal profit have been determined. Perishable products' freshness and greening level can be considered the major elements that influence a buyer's purchasing behaviour. The novelty of the proposed chapter is the concept of greening efforts with freshness and price-related demand is considered. The demand for perishable goods such as green packaged beverages, dairy products, and organic farming products was influenced by consumers' preferences for greenness in addition to freshness and price. Another key point is that perishable goods shouldn't physically or qualitatively degrade more quickly throughout production and that their freshness should be assumed to be 100%. Hence, demand at the beginning of inventory cycle time is price- and green-efforts-dependent. After production stops, products affected by deterioration of both type and product value degrade

continuously, during this period demand pattern depend on freshness, price and greening efforts; Due to freshness degradation, a markdown strategy is adopted after a period of product deterioration to boost demand. Green investment concept in the EPQ model with freshness and markdown strategy, which is another new idea of this study. The results show that markdown offering time and markdown rate make important contributions to maximum total profit, and decision-makers must be very precise when figuring out markdown offering time and markdown rate because the markdown offering should not be too early or too late in order to help in maximising the total profit. The markdown technique is a key method for clearing out stock before it reaches its maximum lifespan. Markdown percentage optimization helps to decision maker to decide the markdown period, hence maximize the profit. The results indicated that more green investments and products with longer shelf lives and shorter deterioration periods boost overall profit with a markdown strategy, which distinguished the previous literature. The problem has been turned into a mathematical model for the purpose of model justification, and a solution process was provided along with an example. To illustrate the analytical results and offer significant managerial implications as a conclusion, we make use of sensitivity analysis.

This model could be extended by taking preservation techniques into account to slow down deterioration. Different payment methods and concepts for carbon policies, such as carbon tax, cap and trade, and carbon limit policies, may be added to the model in the future. Also, by including shortages, the suggested model can be made more inclusive.

CHAPTER-10

Conclusion and Future Research Scope

10.1 Conclusison of the thesis

In this thesis titled '*Modelling Optimal Strategies for Deteriorating Inventory Systems under Different Scenarios*', the research work carried out three different inventory modelling scenarios: an inventory models with a new and used buyback concept; an inventory models with carbon emissions and green investments; and an inventory models that considered product freshness, greening efforts, a price discount, and a markdown strategy. Inventory modelling of new products have a demand pattern that is a non-linear function of selling price and an exponential function of time, a linear function of price and a time-dependent buyback rate, and a demand rate taken for used products. The demand depended on green investments (as a carbon reduction function) and selling price, the promotional level of green investment taken in second scenario models, In the third scenario models, the demand depends on product freshness, greening efforts, and selling price. Trade credit financing policy, selling price discount policy, markdown policy, etc. payment policies adopted to boost market demand.

The inventory models are distinguished in various cases, like shortages not allowed or allowed with partial backlogging, constant and time-dependent deterioration rates, carbon policies, green investments, product freshness, markdown policy etc. The first scenario models are retailer-centric, in which retailers earn revenue by selling new products and used buy-back products while considering deterioration and shortages. Obtained the optimal value of selling price and cycle time such that the retailer's total profit is maximized. The impact of different constant rates of deterioration of new and used products on retailers' profit is evaluated. In the second scenario models, the different sources of carbon emissions to be considered, green technology investment, and their effects on reducing carbon emissions are discussed. Trade credit policy with carbon

policies and green investment strategies are discussed for the protection of the environment and to increase the profit of firms. Developed the sustainable production quantity model for perishable products under different sources of carbon emissions and green investments and optimized the selling price and green investments to maximize the profit of manufacturers. The green VMI and traditional green inventory models are discussed with a green investment strategy under partial backlog shortages. Results show the VMI policy is beneficial in comparison to individual supply chain policy. Perishable product freshness, greening efforts, and selling price require more attention in inventory modelling. A price discount or markdown strategy is to be applied during the deteriorating period to increase the sale of perishable products in the third scenario model. Optimal value of selling price, greening efforts, markdown percentages, markdown offering period, and markdown ordered quantity help to retailer or producer to maximize their profit. All model findings are shown analytically and graphically. The models can be used by operation research and inventory professionals to solve their present manufacturing and stock management challenges.

10.2 Future research directions

Considering the fact that the inventory model's formulation and hypothesis are novel and have not been employed in previous studies, they offer a wide range of prospective studies as a scope of future directions, as below:

- (i) The demand in the present research is deterministic. Demand uncertainty can be anticipated.
- (ii) The rate of deterioration is considered constant and depends on the expiration dates of products; it may be taken as time-dependent stochastic form.
- (iii) Deteriorating products may need proper preservation; future studies may consider preservation technology for a longer life of products.
- (iv) Retiler sells the used products without rework or repairing policy, models may extend with rework policy for imperfect buyback used products.
- (v) In our study developed EOQ and EPQ model with considering carbon emission and green investments, we may extend our study with two or three echlon supply chain model. VMI model developed for single supplier single buyer, one can extend the model with single with vendor multiple buyer or multiple vendor with multiple buyers.

- (vi) Two or three level credit period to be considered as a further study, advance payment policy will be applicable.
- (vii) Possible extensions of each model in this thesis are mentioned in the conclusions section of each chapter.

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List of Publications

➤ Published research articles

1. Katariya DK, Shukla KT. "Retailer's Ordering and Pricing Strategy for New Product and Buyback Strategy for Used Product with Deterioration". *International Journal of Operations Research*. 2021, Jun 1; 18(2):17-28. doi.org/10.6886/IJOR.202106_18(2).0002 (**UGC care listed Journal**)
2. Katariya DK, Shukla KT. "Retailer's optimal inventory strategy for new product and buy back strategy for used product". *Advances and Applications in Mathematical Sciences*. 2021, Sept 1; 20(11): pp.2917-2936. (**UGC care listed Journal**)
3. Katariya D, Shukla K. "Sustainable economic production quantity (SEPQ) model for inventory having green technology investments-price sensitive demand with expiration dates". *Economic Computation & Economic Cybernetics Studies & Research*. 2022 Jul 1; 56(3).pp. 135-152. (**SCI Journal**)
4. Katariya DK, Shukla KT. "An EOQ model for Deteriorating Products with Green Technology Investment and Trade Credit Financing" *Int. J. of Procurement Management*, 2023, Sep 13,18(3): pp.300-320. DOI: 10.1504/IJPM.2023.10048720 (**Scopus Journal**).
5. Katariya DK, Shukla KT. An EPQ Model for Delay Deteriorating Products With Price, Freshness and Greening Efforts Dependent Demand Under Markdown Strategy. *Yugoslav Journal of Operations Research*. 2023 Sep 22. http://dx.doi.org/10.2298/YJOR230515023K (**Scopus Journal**).

➤ Accepted research articles

1. Katariya D, Shukla K. "Pricing and Ordering Strategy for New Product and Buyback Strategy for Used Product from Retailer's Point" *Int. J. of Operational Research*, **Accepted** in August 2021. (**Scopus Journal**).
2. Katariya D, Shukla K. "Retailer's Optimal Pricing and Replenishment Policy for New Product and Buy Back Policy for Used Product with Price and Time Dependent Demand and Shortages". *TARU Journal of Organizational Behaviour & Analytics*, **Accepted** in Feb 2023. (**Peer Review Journal**).

3. Katariya D, Shukla K. “Optimal Greening efforts, Pricing and Inventory Strategies for Non Instantaneous Deteriorating Perishable Products under Price, Freshness and Green efforts Dependent demand with Price Discount”. *Int. J. Of Mathematics in Operational Research*, **Accepted** in April 2023. (**Scopus Journal**).
4. Katariya D, Shukla K.” Optimal Green Investments and Replenishment Decisions in Vendor Managed Inventory System for Non Instantaneous Deteriorating Products with Partially Backordering”. *Int. J. of Business Performance and Supply Chain Modelling*, **Accepted** as per first review and revised article submitted on May,7,2023.(**Scopus Journal**).

➤ **Accepted book chapter**

1. Katariya DK, Shukla KT. “A sustainable inventory models under expiration dates, green technology investments and trade credit policy”. *Supply Chain Finance Modelling and Optimization*, **Accepted** in May 2023. (**Springer Nature Pvt. Ltd.**)

➤ **Research articles presented in conferences**

1. A paper titled, “A Sustainable inventory model of perishable products under expiration dates and green technology investments”, presented in the 4th *International conference on Frontiers Industrial and Applied Mathematics*, organized by SLIET Longowal, Punjab, India, 21-22 December 2021.
2. A paper titled,” Optimal Strategies for Vendor managed Inventory Model with Green investment and it’s promotion level dependent demand and shortages ”, presented in *the International Conference on Applied Mathematical Sciences-2022*, jointly organized by Department of Mathematics, Gujarat University and Department of Applied Science and Humanities, Parul University, Vadodara, Gujarat, India, 12-13 November 2022.